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MATH 251 (04,06,07), Dr. Z. , Final Exam ,Tue., Dec. 19, 2017, SEC 118, 12:00-3:00pm

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question)

Do not write below this line

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1. (out of 12)
2. (out of 12)
3. (out of 12)
4. (out of 12)
5. (out of 12)
6. (out of 12)
7. (out of 12)
8. (out of 12)
9. (out of 12)
10. (out of 12)
11. (out of 12)
12. (out of 12)
13. (out of 12)
14. (out of 12)
15. (out of 12)
16. (out of 12)
17. (out of 8)

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tot. (out of 200)

Important note: Unlike Exams 1 and 2, you are not required to state the type of the answer, and there is no credit for stating the type. But if the given answer is the **wrong type**, you would get 0 points.

Example: Find $f'(2)$ if $f(x) = x^3$. If you give the answer $3x^2$ instead of 12, you would get **zero** points!

Formula that you may (or may not) need

If the surface S is given in **explicit** notation $z = g(x, y)$, above the region of the xy-plane , D, then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \right) dA \quad .$$

1. (12 points) Compute the line-integral

$$\int_C 7y\,dx + 3x\,dy \quad ,$$

where C is the circle $x^2 + y^2 = 100$ traveled in the clockwise direction.

Ans.: Number

$r(t) = \langle 10\cos(t), 10\sin(t) \rangle$

$r'(t) = \langle -10\sin(t), 10\cos(t) \rangle$

$$\int_0^{2\pi} 7(10\sin(t))(-10\sin(t)) + 3(10\cos(t))(10\cos(t))$$

2. (12 points) Find an equation of the tangent plane to the surface

$$z = x^2 + 3xy + y^2 \quad ,$$

at the point $(1, 1, 5)$. $5 = 1 + 3 + 1$

Ans.: Equation

$T = 5 + 5(x-1) + 5(\cancel{x}y-1)$

3. (12 points) Find the absolute maximum value and the absolute minimum value of the function $f(x,y) = x^2\,y$ in the region

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$$\{(x,y)\,|\,0 \le x \le 1\,,\ 0 \le y \le 1-x\,\}.$$

Absolute minimum value: 1

Absolute maximum value: 0

$f_x = 2xy$

$f_{xx} = 2y$

$f_y = x^2$

$f_{yy} = 0$

$f_{xy} = 2x$

$f_y = 0 \Rightarrow x^2 = 0$ cps: $(0,0)$

$f_x = 0 \Rightarrow 2xy = 0$

$D = f_{xx}f_{yy} - f_{xy}^2 = 0$ inconclusive

$f(x,0) = 0$

$f(x,1-x) = x^2(1-x)$ 0, 0

$f(0,y) = 0$

$f(1,y) = y$ = Lx ≤ 0

4. (12 points) Compute $f_{xxyz}(0,0,0)$ (in other words $\frac{\partial^4}{\partial x^2 \partial y \partial z} f(x,y,z)|_{x=0,y=0,z=0}$) if

$$f(x,y,z) = \sin(\, x^2 \; + \; y \; + \; z\,) \quad .$$

Ans.: Number

$F_x = \cos(x^2+y+z) \cdot 2x$

$f_{xx} = 2\cos(x^2+y+z) - 2x\sin(x^2+y+z) \cdot 2x$

$f_{xxy} = -2\sin(x^2+y+z) - 4x^2\cos(x^2+z+y)$

$f_{xxyz} = -2\cos(x^2+y+z) + 4x^2\sin(x^2+z+y)$

5. (12 points) Find $\frac{\partial z}{\partial y}$ at the point $(1, 1, 1)$ if (x, y, z) are related by:

$$xy + xz + yz + x^2y^2z^2 = 4 \quad .$$

Ans.: Number

$$x + x\frac{\delta z}{\delta y} + z + y\frac{\delta z}{\delta y} + (xy)^2(2z)\frac{\delta z}{\delta y} + z^2(2x^2y) = 0$$

$$\frac{-x - z - 2x^2yz^2}{x + y + 2x^2y^2z} = \left.\frac{\delta z}{\delta y}\right|_{1,1,1} = \frac{-1 - \cancel{1} - 2}{1 + 1 + 2} = -1$$

6. (12 points) Find an equation for the plane that contains both the line

$$x = 1 + t\,,\; y = 2 + t\,,\; z = 3 + t \quad (-\infty < t < \infty) \quad ,$$

and the line

$$x = -t\,,\; y = 1 + t\,,\; z = 2 + t \quad (-\infty < t < \infty) \quad .$$

Ans.: Equation

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7. (12 points) A certain particle has acceleration given by

$$\mathbf{a}(t) = \langle\, -4\sin 2t\,,\; -4\cos 2t\,,\; 9e^{3t}\,\rangle \quad .$$

If its velocity at $t = 0$ is $\langle 2, 0, 3\rangle$ and its position at $t = 0$ is $\langle 0, 1, 1\rangle$, finds its position at the time $t = \frac{\pi}{4}$.

Ans.:

$a(t) = \langle -4\sin(2t), -4\cos(2t), 9e^{3t} \rangle$

$V(t) = \langle 2\cos(2t), -2\sin(2t), 3e^{3t} \rangle$ $\quad v(0) = \langle 2, 0, 3 \rangle$

$\int -4\sin(2t)dt$; $u = 2t$, $du = 2dt$; $\frac{1}{2}\int -4\sin(u)\,du \Rightarrow -2\int \sin(u) \overset{du}{=} 2\cos(u) + C = 2\cos(2t) + C_1$; $C_1 = 0$

$\int -4\cos(2t)dt$; $u = 2t$, $du = 2dt$; $\frac{1}{2}\int -4\cos(u)\,du \Rightarrow -2\int \cos(u)\,du = -2\sin(u) + C = -2\sin(2t) + C_2$; $C_2 = 0$

$\int 9e^{3t}dt$; $u = 3t$, $du = 3dt$; $3\int e^u du \Rightarrow 3e^u + C \Rightarrow 3e^{3t} + C_3$; $C_3 = 0$

$r(t) = \langle \sin(2t), \cos(2t), e^{3t} \rangle$ $\quad r(0) = \langle 0, 1, 1 \rangle$

$\int 2\cos(2t)\,dt$; $u = 2t$, $du = 2dt$; $\int \cos(u)du = \sin(u) + C_1$; $C_1 = 0$

$\int -2\sin(2t)$; $u = 2t$, $du = 2dt$; $-\int \sin(u) \overset{du}{=} \cos(u) + C_2$; $C_2 = 0$

$\int 3e^{3t} \Rightarrow e^{3t} + C_3$; $C_3 = 0$

$V(\frac{\pi}{4}) = \langle 0, -2, 3e^{3\pi/4} \rangle$

$r(\frac{\pi}{4}) = \langle 1, 0, e^{\frac{3\pi}{4}} \rangle$

8. (12 points) Compute the (scalar-function) line-integral

$$\int_C (x + y + 2z)\, ds$$

where the curve C is given by the parametric equation:

$$\mathbf{r}(t) = \langle t, 2t, 2t \rangle \quad , \quad 0 \le t \le 1 \quad .$$

Ans.: Number

$$\int_0^1 (t + 2t + 4t)\, 3\, dt$$

9. (12 points)

If

$$\lim_{(x,y,z)\to(1,1,1)} f(x,y,z) = 1 \quad , \quad \lim_{(x,y,z)\to(1,1,1)} g(x,y,z) = 2$$

compute

$$\lim_{(x,y,z)\to(1,1,1)} \sin(\frac{\pi}{3} f(x,y,z)) \cos(\frac{\pi}{4} g(x,y,z))$$

Ans.: 0

$$\lim_{(x,y,z)\to(1,1,1)} \sin\left(\frac{\pi}{3}\right) \cdot \lim_{x,y,z} \cos\left(\frac{\pi}{2}\right) = 0$$

10. (12 points) Compute

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} \quad ,$$

where

$$\mathbf{F} = \langle x^2 + \sin(y+z)\,,\, y^2 + xz^3\,,\, z^2 + e^{xy} \rangle$$

and where S is the boundary (consisting of all six faces) of the cube

$$\{(x, y, z) \,|\, 0 \leq x, y, z \leq 1\}$$

with the normal pointing **outward**.

Ans.:

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11. (12 points) By finding a function f such that $\mathbf{F} = \bigtriangledown f$, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.

$$\mathbf{F}(x, y, z) \ = \ \langle\, 2e^{2x+3y+4z} \,,\, 3e^{2x+3y+4z} \,,\, 4e^{2x+3y+4z} \,\rangle \quad ,$$

$$C : x = t \quad , \quad y = 2t \quad , \quad z = t^2 \quad , \quad 0 \leq t \leq 1 \quad .$$

Ans:

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12. (12 points) Evaluate the line integral

$$\int_C 5y\,dx + 5x\,dy + 6z\,dz \quad ,$$

where $C : x = t^2\,,\ y = t\,,\ z = t^2\,,\ 0 \leq t \leq 1.$

Ans.: Number

$$\int_0^1 \left[5t\,(2t) + 5t^2 + 6t^2(2t)\right] dt$$

13. (12 points) Evaluate

$$\int\int\int_E \frac{1}{\sqrt{x^2+y^2+z^2}}\, dV \quad ,$$

where E is the hemisphere

$$\{(x, y, z) \,|\, x^2 + y^2 + z^2 \leq 100, z < 0\} \quad .$$

$$\{(\rho, \theta, \phi) \,|\, 0 \leq \rho \leq 10,\ 0 \leq \theta \leq 2\pi,\ 0 \leq \phi \leq \tfrac{\pi}{2}\}$$

Ans.: 100π

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{10} \rho \sin\phi \, d\rho\, d\phi\, d\theta \Rightarrow ①\ \int_0^{10} \rho \sin\phi \, d\rho = \int_0^{\frac{\pi}{2}} 50 \sin\phi \, d\phi =$$

$$-50\cos\phi \Big|_0^{\frac{\pi}{2}} = \int_0^{2\pi} 50 \, d\theta = \quad 100\pi$$

14. (12 points) Evaluate the quadruple integral

$$\int\int\int\int_E 360\,x\,dV \quad ,$$

where

$$E = \{(x,y,z,w)\,|\,0 \le w \le 1\,,\ 0 \le z \le w\,,\ 0 \le y \le z\,,\ 0 \le x \le y\} \quad .$$

Ans.: 3

$$\int_0^1\int_0^w\int_0^z\int_0^y 360x\,dx\,dy\,dz\,dw$$

$$180x^2\Big|_0^y = \int_0^z 180y^2\,dy \qquad 60y^3\Big|_0^z = \int_0^w 60z^3\,dz$$

$$15z^4\Big|_0^w = 15w^4 \Rightarrow \int_0^1 15w^4\,dw = \boxed{3}$$

15. (12 points) Find the Jacobian of the transformation from (u, v)-space to (x, y)-space.

$$x = 3\sin(2u + v) \quad , \quad y = u + v + \cos(u + v) \quad ,$$

at the point $(u, v) = (0, 0)$.

Ans.: Number 3

$$\begin{vmatrix} 6\cos(2u+v) & 3\cos(2u+v) \\ 1-\sin(u+v) & 1-\sin(u+v) \end{vmatrix} = \begin{vmatrix} 6 & 3 \\ 1 & 1 \end{vmatrix}_{u=0,\, v=0}$$

$6 - 3 = 3$

16. (12 points) Find the local maximum and minimum **points** and saddle point(s) of the function $f(x, y) = x^3 + y^2 - 6xy$

Local maximum points(s): none

Local minimum points(s): none

saddle point(s): (0,0) (6,18)

$f_x = 3x^2 - 6y$

$f_{xx} = 6x$

$f_y = 2y - 6x$

$f_{yy} = 2$

$f_{xy} = -6$

$f_x = 0 \Rightarrow 3x^2 - 6y = 0 \Rightarrow \cancel{3}x^2 - 6(\cancel{3}x) = 0$ $\quad x(x-6) = 0$

$x = 0, 6$

$f_y = 0 \Rightarrow 2y - 6x = 0 \Rightarrow y = 3x$ $\quad y = 0, 18$

CPs: $\underset{P}{(0,0)}$, $\underset{Q}{(6,18)}$

$D_P = f_{xx}(P) \cdot f_{yy}(P) - f_{xy}(P)^2 = -36$ saddle @ P

$D_Q = f_{xx}(Q) \cdot f_{yy}(Q) - f_{xy}^2(Q) = -36$ saddle @ Q

$6^2 \cdot \left(2(18) - 6(6)\right) - f_{xy}$

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17. (8 points) Use the Divergence Theorem to calculate the surface integral $\int \int_S \mathbf{F} \cdot d\,\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = \langle\, x + y \;\; y + z \,,\, x + z \,\rangle \quad ,$$

where S is the sphere (center $(1, -2, 4)$ and radius 10), in other words the region in 3D space:

$$\{(x, y, z) \,|\, (x-1)^2 \,+\, (y+2)^2 \,+\, (z-4)^2 = 100\} \quad .$$