1. $x = 10\cos t \qquad dx = -10\sin t\,dt$
   $y = 10\sin t. \qquad dy = 10\cos t\,dt$

$$\int_0^{2\pi} 70\sin t \cdot (-10\sin t)\,dt + 30\cos t \cdot (10\cos t)\,dt$$

$$= \int_0^{2\pi} (-700\sin^2 t + 300\cos^2 t)\,dt$$

$$= 400\pi.$$

2. $1 + 3\times1\times1 + 1 = 5.$

$g(x,y) = \langle 2x+3y,\ 2y+3x \rangle$

2. $g(1,1) = \langle 5, 5 \rangle$

$z - 5 = 5(x-1) + 5(y-1).$
$\quad = 5x + 5y - 10$

$5x + 5y - z = 5.$

3. $g(x,y) = \langle 2xy,\ x^2 \rangle$

$\because x^2 \geq 0,\ y \geq 0$
$f(x,y) \geq 0.$
$x = y = 0,$
$f(x,y) = 0$ is the absolute minimum.

$g(x,y) = x^2(1-x)$

$g'(x,y) = 2x - 3x^2 = 0$

$x_1 = 0 \quad x_2 = \frac{2}{3}$

$\left(\frac{2}{3}\right)^2 \times \left(1 - \frac{2}{3}\right) = \frac{4}{27}$

$\therefore f(x,y) = \frac{4}{27}$ is the absolute maximum.

4. $f_x = 2x\cos(x^2+y+z)$

$f_{xx} = 2\cos(x^2+y+z) - 4x^2\sin(x^2+y+z)$.

$f_{xxy} = -2\sin(x^2+y+z) - 4x^2\cos(x^2+y+z)$.

$f_{xxyz} = -2\cos(x^2+y+z) + 4x^2\sin(x^2+y+z)$

$f_{xxyz}(0,0,0) = -2+0 = -2$.

5. $x\frac{\partial z}{\partial y} + x + z\frac{\partial z}{\partial y} + y + x^2(2yz^2\frac{\partial z}{\partial y} + 2y^2z) = 0$.

when $(x,y,z) = (1,1,1)$.

$4\frac{\partial z}{\partial y} + 4 = 0$

$\frac{\partial z}{\partial y} = -1$.

6. $L_1 = \langle t+1, t+2, t+3\rangle$. $g_1 = \langle 1,1,1\rangle$

$L_2 = \langle -t, t+1, t+2\rangle$. $g_2 = \langle -1,1,1\rangle$.

$\cancel{L_1 \times L_2 = \langle 1, -2t^2-6t-2, 2t^2+4t+1\rangle}$.

~~Assume t=0~~ $g_1 \times g_2 = \langle 0,-2,2\rangle$.

$\cancel{L_1 \times L_2 = \langle 1,}$ Assume $t=0$.

The point is $(1,2,3)$

$-2(y-2)+2(z-3)=0$

$-2y+2z-2=0$

$y-z+1=0$.

7. $V(t) = \langle 2\cos 2t, -2\sin 2t, 3e^{3t} \rangle + C$.

~~pass~~ ~~[illegible]~~

$V(0) = \langle 2, 0, 3 \rangle$.

$\therefore V(t) = \langle 2\cos 2t, -2\sin 2t, 3e^{3t} \rangle$.

$S(t) = \langle \sin 2t, \cos 2t, e^{3t} \rangle + C$.

$S(0) = \langle 0, 1, 1 \rangle$.

$\therefore S(t) = \langle \sin 2t, \cos 2t, e^{3t} \rangle$.

$S(\frac{\pi}{4}) = \langle 1, 0, e^{\frac{3}{4}\pi} \rangle$.

8. $r'(t) = \langle 1, 2, 2 \rangle$.

$ds = \sqrt{2^2+2^2+1^2}\, dt = 3dt$.

$3\int_0^1 (t+2t+4t)\,dt = 21 \times \frac{1}{2} = \frac{21}{2}$

9. $\sin(\frac{\pi}{3}) \cdot \cos(\frac{\pi}{2}) = \frac{\sqrt{3}}{2} \times 0 = 0$.

10. $\mathrm{div}(F) = \cancel{\langle 2x, 2} \; 2x+2y+2z$.

$\int_0^1 \int_0^1 \int_0^1 (2x+2y+2z)\,dx\,dy\,dz = 3$.

11. For x, $f(x,y,z) = e^{2x+3y+4z} + g(y,z)$.

Add For y, $f(x,y,z) = e^{2x+3y+4z} + h(z)$.

Add For z $f(x,y,z) = e^{2x+3y+4z}$

$t=0$, $(x,y,z) = (0,0,0)$.

$t=1$, $(x,y,z) = (1,2,1)$.

$\int_C F \cdot dr = f(1,2,1) - f(0,0,0) = e^{12} - 1$.

12. $\begin{cases} dx = 2t\,dt \\ dy = dt \\ dz = 2t\,dt. \end{cases}$

$\int_0^1 (5t)(2t\,dt) + (5t^2)(dt) + (6t^2)(2t\,dt)$

$= \int_0^1 (15t^2 + 12t^3)\,dt$

$= (5t^3 + 3t^4)\Big|_0^1$

$= 8$

13. $\int_0^{10} \int_{\frac{\pi}{2}}^{\pi} \int_0^{2\pi} \rho \sin\phi \, d\theta \, d\phi \, d\rho = 100\pi$.

14. $\int_0^1 \int_0^w \int_0^z \int_0^y 360x \, dx\, dy\, dz\, dw = 3$.

15. $g_x = \langle 6\cos(2u+v), 3\cos(2u+v) \rangle$ $g_x(0,0) = \langle 6, 3 \rangle$

$g_y = \langle 1-\sin(u+v), 1-\sin(u+v) \rangle$. $g_y(0,0) = \langle 1, 1 \rangle$.

$6 \times 1 - 3 \times 1 = 3$.

16. $g(x,y) = \langle 3x^2 - 6y, 2y - 6x \rangle$.

$$\begin{cases} 3x^2 - 6y = 0 \\ 2y - 6x = 0 \end{cases}$$

$x_1 = 0, x_2 = 6$

$(0,0)$, $(6,18)$ are critical points.

For $(0,0)$. $D = \frac{d}{dx}(3x^2) \cdot \frac{d}{dy}(2y) - \left[\frac{d}{dy}(-6y)\right]^2 = 12x - 36 = -36 < 0$

$\therefore$ It's saddle point.

For $(6,18)$ $D = 12x - 36 = 36 > 0$.

$\frac{d}{dx}(3x^2) = 36 > 0$.

$\therefore$ It's local minimum.

There's no local maximum.

17. $\text{div}(F) = 1+1+1 = 3$.

$3 \times \frac{4\pi}{3} \cdot 10^3 = 4000\pi$.