Calc Final Review

1. $\int_C 7y\,dx + 3x\,dy$ where C is the circle $x^2+y^2=100$ traveled in ↺

$y = 10\sin t \quad x = 10\cos t$

$\frac{dy}{dt} = 10\cos t \quad \frac{dx}{dt} = -10\sin t \qquad 0 \le t \le 2\pi$

$\int_0^{2\pi} (7(10\sin t) + 3(10\cos t))\cdot(10\,dt) \quad \Big| \quad \sqrt{100\cos^2 t + 100\sin^2 t} = 10\,dt$

$10\int_0^{2\pi} 70\sin t + 30\cos t\,dt = \boxed{0}$

2. $z = x^2+3xy+y^2$ at point $(1,1,5)$ $\qquad x^2+3xy+y^2-z=0$

$g(x,y,z) = x^2+3xy+y^2-z \qquad g_x = 2x+3y,\ g_y = 3x+2y,\ g_z = -1$

$g_x(1,1,5) = 5 \quad g_y(1,1,5) = 5 \quad g_z = -1$

$g_x(x-x_0) + g_y(y-y_0) + g_z(z-z_0) = 0 \qquad 5(x-1)+5(y-1)-(z-5)=0$

$\boxed{z = 5x+5y-5}$

5. Find $\frac{dz}{dy}$ at the point $(1,1,1)$ if (x,y,z) are related by

$xy + xz + yz + x^2y^2z^2 = 4$

$F_y = x+z+2x^2yz^2 \quad F_z = x+y+2x^2y^2z \quad \frac{dz}{dy} = \frac{-F_y}{F_z} \quad F_y @ (1,1,1)$

$F_y(1,1,1) = 4$

$F_z(1,1,1) = 4$

$\frac{dz}{dy} = -\frac{4}{4} = \boxed{-1}$

4. compute $f_{xxyz}(0,0,0)$ if $f(x,y,z) = \sin(x^2+y+z)$

$f(x,y,z) = \sin(x^2+y+z) \quad f_x = 2x\cos(x^2+y+z)$

$f_{xx} = 2\cos(x^2+z+y) - 4x^2\sin(x^2+z+y) \quad f_{xxy} = -2(\sin(x^2+z+y) + 2x^2\cos(x^2+y+z))$

$f_{xxyz} = 2(\sin(x^2+z+y) - 2x^2\cos(x^2+y+z))$

$f_{xxyz}(0,0,0) = \boxed{0}$

7. $a(t) = \langle -4\sin 2t, -4\cos 2t, 9e^{3t} \rangle$

$v(t) = \int a(t)\,dt = \langle 2\cos(2t)+2, -2\sin(2t), 3e^{3t}+3 \rangle$

$x(t) = \int v(t)\,dt = \langle \sin(2t)+2t, \cos(2t)+1, e^{3t}+3t+1 \rangle$

$\boxed{x(\frac{\pi}{4}) = \langle 1+\frac{\pi}{2}, 1, e^{3\pi/4}+\frac{3\pi}{4}+1 \rangle}$

8. $\int_C (x+y+2z)\,ds \quad r(t) = \langle t, 2t, 2t \rangle, \quad 0 \le t \le 1$

$f(x,y,z) = x+y+2z \quad f(r(t)) = t+2t+4t = 7t \quad r'(t) = \langle 1,1,2 \rangle$

$|r'(t)| = \sqrt{6}$

$\int_0^1 7\sqrt{6}\,t\,dt = \boxed{\frac{7\sqrt{6}}{2}}$

6. $n_1 = \langle 1,1,1 \rangle, \; n_2 = \langle -1,1,1 \rangle \quad n_1 \times n_2 = \langle 0,-2,2 \rangle \quad N = \langle 0,-2,2 \rangle$

$N_1(x-x_0) + N_2(y-y_0) + N_3(z-z_0) \quad @\ t=0: P=(1,2,3) \; Q=(0,1,2)$

$-2(y-2) + 2(z-3) = 0 \quad -2y+4+2z-6=0 \quad \boxed{y-1=z}$

9. $\lim_{(x,y,z)\to(1,1,1)} \left(\sin(\frac{\pi}{4}) \cdot \cos(\frac{\pi}{2}) \right) = \frac{\sqrt{2}}{2} \cdot 0 = \boxed{0}$

3. $\{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1-x\}$ $\quad f(x,y) = x^2 y \quad f_x = 2xy \quad f_y = x^2$

$2xy = 0 \quad x^2 = 0$

$x=0, \; (0,y) \quad f(0,y) = 0$ (for y in $[0, 1-x]$) $\quad$ cp @ $(0,1)$ $\quad f(0,1) = 0$

abs min = 0 abs max = 0

$y=0, \; (x,0) \quad f(x,0) = 0$ (for x in $[0,1]$)

abs min = 0 abs max = 0

$y = 1-x \quad (x, 1-x) \quad f(x,1-x) = x^2(1-x)$ * for x in $[0,1]$

$g(x) = x^2(1-x) = x^2 - x^3 \quad g(0)=0 \quad g(0)=0$

$g'(x) = 2x - 3x^2 = 0 \quad g(1) = 0 \quad g(2/3) = 0$

$\boxed{\text{Abs min} = 0 \quad \text{Abs max} = 0}$

14. $\iiiint_E 360x\,dV$ $\int_0^1\int_0^4\int_0^x\int_0^y 360x\,dV = \boxed{15}$

15. $x = 3\sin(2u+v)$ $y = u+v+\cos(u+v)$ $(u,v) = (0,0)$

$\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$ $x_u = 6\cos(2u+v)$ $y_u = 1-\sin(v+u)$ $x_u = 6$ $y_u = 1$

$x_v = 3\cos(2u+v)$ $y_v = 1-\sin(v+u)$ $x_v = 3$ $y_v = 1$

$J = x_u y_v - x_v y_u$ $J = 6-3$ $\boxed{J=3}$

10. $\operatorname{div} F = \frac{d}{dx}(x^2+\sin(y+z)) + \frac{d}{dy}(y^2+xz^3) + \frac{d}{dz}(z^2+e^{xy}) = 2x+2y+2z$

$\int_0^1\int_0^1\int_0^1 (2x+2y+2z)\,dz\,dy\,dx = \boxed{3}$

11. $\int 2e^{2x+3y+4z} = e^{2x+3y+4z} + g(y,z)$

$f = e^{2x+3y+4z}$ $r(t): \langle t, 2t, t^2 \rangle$ $r(0) = \langle 0,0,0 \rangle$ $r(1) = \langle 1,2,1 \rangle$

$f(1,2,1) - f(0,0,0) = e^{12} - e^0 = \boxed{e^{12}-1}$

12. $\int_0^1 (10t^2+5t^2+12t^3)\,dt = \int_0^1 (15t^2+12t^3)\,dt = 8$

13. $\frac{\pi}{2} \leq \varphi \leq \pi$; $0 \leq \theta \leq 2\pi$; $0 \leq \rho \leq 10$ $\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_0^{10} \rho\sin\varphi\,d\rho\,d\varphi\,d\theta = \boxed{100\pi}$

16. $f(x,y) = x^3+y^2-6xy$ $f_x = 3x^2-6y$ $f_y = 2y-6x$ $f_{xx} = x$ $f_{xy} = 4$ $f_{xy} = -6$

cp @ $(0,0)$ and $(6,18)$ -36 ; $D<0$ for $(0,0)$: saddle point

108 ; $D>0$ for $(6,18)$: local min

17. $\operatorname{div}(F) = 3$ $\iiint 3\,dV$ $\frac{4}{3}\pi R^3 \rightarrow V = 4000\pi$