Lecture 25 Quiz Shaun Goda

January February March April May June July August September October November December
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

1)

$$\text{Curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos(\sqrt{1+x^7}+zy^9) & \tan(x^7+y^2+\frac{1}{z}) & \tan(e^{xyz}+\cos(x^8-y+3z)) \end{vmatrix}$$

$$= ($$

$$\text{div } F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\text{div } \nabla \times F = 0$$

answer: 0

2) Calculate the surface integral... $z = g(x,y) = 5$

$$S_1 = \iint_S F \cdot dS = \iint_D \left(-(2x+y+z)0 - (x+2y+z)0 + (x+y+2z)\right) dA$$

$$\iint_D (x+y+2z)\,dA = \iint_D (x+y+10)\,dA$$

where $\{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 4\}$

$$\int_0^4 \int_0^1 (x+y+10)\,dx\,dy$$

$$\Rightarrow \int_0^1 (x+y+10)\,dx = \left| \frac{x^2}{2} + xy + 10x \right|_0^1 = \frac{1}{2} + y + 10$$

$$\int_0^4 (y + \tfrac{21}{2})\,dy = \left| \frac{y^2}{2} + \frac{21}{2}y \right|_0^4 = 8 + 22 = 30$$

$$S_2 = \int_0^4 \int_0^1 (x+y)\,dx\,dy \Rightarrow \int_0^1 (x+y)\,dx = \left| \frac{x^2}{2} + xy \right|_0^1 = \frac{1}{2} + y$$

$$\int_0^4 (\tfrac{1}{2} + y)\,dy = \left| \frac{1}{2}y + \frac{y^2}{2} \right|_0^4 = 2 + 8 = 10$$

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$S_3 = \iint_S F \cdot dS = \iint_D \left(-P - Q\frac{\partial g}{\partial y} + R\frac{\partial g}{\partial z}\right) dA$ $\quad x = g(y,z) = 1$

$= \iint_D (-(2x+y+z))\, dA = \int_0^5 \int_0^4 (-2 - y - z)\, dy\, dz$

$\Rightarrow \int_0^4 (-2-y-z)\, dy = \left| -2y - \frac{y^2}{2} - zy \right|_0^4$

$= -8 - 8 - 4z = -16 - 4z$

$\int_0^5 (-16-4z)\, dz = \left| -16z - 2z^2 \right|_0^5 = -80 - 50 = \underline{-130}$

$S_4 = \int_0^5 \int_0^4 (-y-z)\, dy\, dz \Rightarrow \int_0^4 (-y-z)\, dy = \left| -\frac{y^2}{2} - zy \right|_0^4$

$= -8 - 4z \Rightarrow \int_0^5 (-8-4z)\, dz = \left| -8z - 2z^2 \right|_0^5$

$= -40 - 50 = \underline{-90}$

$S_5 = \iint_S F \cdot dS = \iint_D \left(-P\frac{\partial g}{\partial x} - Q + R\frac{\partial g}{\partial z}\right) dA \quad y = g(x,z) = 4$

$= \iint_D (-(x+2y+z))\, dA = \int_0^5 \int_0^1 (-x+8+z)\, dx\, dz$

$\Rightarrow \int_0^1 (-x+8+z)\, dx = \left| -\frac{x^2}{2} + 8x + xz \right|_0^1 = -\frac{1}{2} + 8 + z$

$\int_0^5 \left(-\frac{15}{2} + z\right) dz = \left| -\frac{15}{2}z + \frac{z^2}{2} \right|_0^5 = -\frac{75}{2} + \frac{25}{2} = \underline{-25}$

$S_6 = \int_0^5 \int_0^1 (-x+z)\, dx\, dz \Rightarrow \int_0^1 (-x+z)\, dx = \left| -\frac{x^2}{2} + zx \right|_0^1$

$= -\frac{1}{2} + z \Rightarrow \int_0^5 \left(-\frac{1}{2} + z\right) dz = \left| -\frac{1}{2}z + \frac{z^2}{2} \right|_0^5$

$= -\frac{5}{2} + \frac{25}{2} = \frac{20}{2} = \underline{10}$

$|\Sigma S_n| = 30 + 10 - 130 - 90 - 25 + 10$

$= -195$

answer : -195