

"QUIZ" for Lecture 25

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q25FirstLast.pdf) ASAP BUT NO LATER THAN Dec.8,2020, 8:00pm

Let

$$F(x, y, z) =$$

$$\langle \cos(\sqrt{1+x^7+zy^9}), \tan(x^7+y^2+1/z), \tan^{-1}(e^{xyz} + \cos^6(x^8-y+3z)) \rangle,$$

and let $\langle P, Q, R \rangle = \text{curl } \mathbf{F}$. Compute

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$$

Be sure to explain everything.

$$\left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) = \text{div} \langle P, Q, R \rangle = \text{div}(\text{curl}(F))$$

$$\text{div}(\text{curl}(F)) = 0$$

for any differentiable F

2. Calculate the surface integral

$\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = \langle 2x + y + z, x + 2y + z, x + y + 2z \rangle$$

where S is the surface of the box bounded by the planes $x = 0, x = 1, y = 0, y = 4, z = 0, z = 5$.

$$\text{div}(F) = 2 + 2 + 2 = 6 \quad \iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div } F \, dV$$

$$= \int_0^5 \int_0^4 \int_0^1 6 \, dx \, dy \, dz = \boxed{120}$$