

Quiz 25.

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Section: 23

1. Let $F(x, y, z) = \langle \cos(\sqrt{x^2+1} + zy^9), \tan(x^7 + y^2 + \frac{1}{z}), \tan^{-1}(e^{xyz} + \cos^6(x^8 - y + 3z)) \rangle$

and let $\langle P, Q, R \rangle = \text{curl } F$.

Compute $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

Be sure to explain everything

$$\textcircled{1} \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = \text{div}(\text{curl } F) = 0.$$

2. Calculate the surface integral

$\iint_S F \cdot dS$, where

$$F(x, y, z) = \langle zx + y + z, x + zy + z, x + y + 2z \rangle$$

where S is the surface of the box bounded by the planes $x=0$, $x=1$, $y=0$, $y=4$, $z=0$, $z=5$.

$$\text{div}(F) = 2 + 2 + 2 = 6.$$

$$\int_0^1 \int_0^4 \int_0^5 6 \, dz \, dy \, dx$$

$$= 6 \times 5 \times 4$$

$$= 120$$

