"QUIZ" for Lecture 25

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q25FirstLast.pdf) ASAP BUT NO LATER THAN Dec.8,2020, 8:00pm

Let

$$F(x,y,z) =$$

$$\langle \cos(\sqrt{1+x^7}+zy^9) \quad , \quad \tan(x^7+y^2+1/z) \quad , \quad \tan^{-1}(e^{xyz}+\cos^6(x^8-y+3z)) \rangle \quad ,$$

and let $\langle P, Q, R\rangle = curl\ \mathbf{F}$. Compute

$$\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z} \quad .$$

Be sure to explain everything.

$\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}$ is also a formula for $div(F)$, if $F = \langle P, Q, R\rangle$

If we take curl of F, we get:

$curl(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \hat{i}(R_y - Q_z) - \hat{j}(R_x - P_z) + \hat{k}(Q_x - P_y)$

And taking divergence of the result is:

$div(curl(F)) = \frac{\partial}{\partial x}(R_y - Q_z) - \frac{\partial}{\partial y}(R_x - P_z) + \frac{\partial}{\partial z}(Q_x - P_y) =$

$= R_{yx} - Q_{zx} - R_{xy} + P_{zy} + Q_{xz} - P_{yz} = (P_{zy} - P_{yz}) + (Q_{xz} - Q_{zx}) + (R_{yx} - R_{xy}) = 0$

So, $\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}$ of $curl(F)$ is $\boxed{0}$

2. Calculate the surface integral
$\int\int_S \mathbf{F}\cdot d\mathbf{S}$, where

$$\mathbf{F}(x,y,z) = \langle 2x+y+z\, ,\, x+2y+z\, ,\, x+y+2z\rangle$$

where S is the surface of the box bounded by the planes $x=0$, $x=1$, $y=0$, $y=4$, $z=0$, $z=5$.

Divergence theorem states that:

$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E div\,\vec{F}\,dV$, where E is our region in the question.

$div(F) = \frac{\partial}{\partial x}(2x+y+z) + \frac{\partial}{\partial y}(x+2y+z) + \frac{\partial}{\partial z}(x+y+2z) = 2+2+2 = 6$

So, our new integral is:

$\iiint_E 6\,dV$

Our region E would be a box, where $0 \le x \le 1$, $0 \le y \le 4$, and $0 \le z \le 5$. The volume integral of a constant is the volume of the region times the constant. The volume of the box is $1 \cdot 4 \cdot 5 = 20 \Rightarrow 20 \cdot 6 = \boxed{120}$