Quiz for Lecture 25

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① Let

$$F(x,y,z) =$$

$$\langle \cos\left(\sqrt{1+x^2}+2y^9\right),\ \tan\left(x^7+y^3+\frac{1}{z}\right),\ \tan^{-1}\left(e^{xyz}+\cos(k^8 y+3z)\right)\rangle,$$

and Let $\langle P,Q,R\rangle = \operatorname{curl} F$. Compute.

$$\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}.$$

Be sure to explain everything.

Ans: 0.

$\operatorname{curl} F$:

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$i(R_y - Q_z) - j(R_x - P_z) + k(Q_x - P_y).$$

$$\langle R_y - Q_z,\ -R_x + P_z,\ Q_x - P_y\rangle$$

$\operatorname{div}(\operatorname{curl} F)$:

$$R_{yx} - Q_{zx} - R_{xy} + P_{zy} + Q_{xz} - P_{yz}.$$

$$= 0.$$

Ans: 0.

② Calculate the surface integral $\iint_S F \cdot dS$, where

$$F(x,y,z) = \langle 2x+y+z,\ x+2y+z,\ x+y+2z \rangle.$$

where S is the surface of the box bounded by the planes $x=0$, $x=1$, $y=0$, $y=4$, $z=0$, $z=5$.

$$\iint_S F \cdot dS = \iiint_V \operatorname{div} F \, dV$$

$$\operatorname{div} F = 2 + 2 + 2 = 6.$$

$$\iiint_V 6 \, dV = 6 \times \text{volume} = 6 \times 1 \times 4 \times 5$$
$$= 6 \times 20$$
$$= 120.$$

Ans: 120.