Yash Khangura "Quiz" for Lecture 24 Section 24

By using Stokes' Theorem, or otherwise, evaluate $\int_C F \cdot dr$, where C is the curve if intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=1$, oriented counterclockwise as viewed from above. Be sure to explain everything.

$$F(x,y,z) = (yz + 2y + 3z)i + (xz + 2x + 4z)j + (xy + 3x + 4y)k$$

$$r = \langle v\cos u, v\sin u, 1 - v\cos u - v\sin u \rangle$$

$$r_u = \langle -v\sin u, v\cos u, 1 + v\sin u - v\cos u \rangle$$

$$r_v = \langle \cos u, \sin u, -\cos u - \sin u \rangle$$

$$r_u \times r_v = \langle (-v\cos^2 u \cancel{- v\cos u\sin u} - \sin u - v\sin^2 u \cancel{+ v\cos u \sin u}),$$
$$(\cos u \cancel{+ v\cos u\sin u} - v\cos^2 u \cancel{- v\cos u\sin u} - v\sin^2 u), -v\sin^2 u - v\cos^2 u \rangle$$

$$= \langle -v - \sin u, -v + \cos u, -v \rangle$$

$$= \langle v + \sin u, v - \cos u, v \rangle$$

We use the simplest surfaces $x+y+z=1$ inside the cylinder and find r. We find $r_u \times r_v$ and multiply by -1 to match the orientation of counterclockwise.

$$\text{curl } F = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz + 2y + 3z & xz + 2x + 4z & xy + 3x + 4y \end{vmatrix}$$

$$= \langle (x+4) - (x+4), (y+3) - (y+3), (z+2) - (z+2) \rangle = \langle 0, 0, 0 \rangle$$

Wow, this was a trick question. F is a conservative vector field, so the answer is just 0.