"Quiz" for Lecture 24 - Shaun Goda

By using stoke's theorem, or otherwhise, evaluate $\int_C F \cdot dr$, where

$$F(x, y, z) = (yz + 2y + 3z)i + (xz + 2x + 4z)j + (xy + 3x + 4y)k$$

where C is the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$ oriented counterclockwise as viewed from above. Be sure to explain everything.

$$g(x, y) = 1 - x - y$$

$$D = \{(x, y) \mid -1 \leq x \leq 1, -1 \leq y \leq 1\}$$

$$\text{Curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz + 2y + 3z & xz + 2x + 4z & xy + 3x + 4y \end{vmatrix}$$

$$= ((x + \overset{2}{\cancel{4}}) - (x + \overset{2}{\cancel{4}}))i - ((y + \overset{3}{\cancel{4}}) - (y + 3))j + ((z + \overset{2}{\cancel{4}}) - (z + 2))k$$

$$= \langle 0, 0, 0 \rangle$$

$$\int_C F \cdot dr = \int_S \text{curl} F \cdot dS = \int_S \langle 0, 0, 0 \rangle dS = 0$$

answer is 0