

"QUIZ" for Lecture 24

NAME: (print!) SAI EMBAR Section: 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q24FirstLast.pdf) ASAP BUT NO LATER THAN Dec. 4, 2020, 8:00pm

By using Stokes' Theorem, or otherwise, evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where

$$\mathbf{F}(x, y, z) = (yz + 2y + 3z)\mathbf{i} + (xz + 2x + 4z)\mathbf{j} + (xy + 3x + 4y)\mathbf{k}$$

where  $C$  is the curve of intersection of the plane  $x + y + z = 1$  and the cylinder  $x^2 + y^2 = 1$ , oriented counterclockwise as viewed from above. Be sure to explain everything.

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

$$\text{curl}(\mathbf{F}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz + 2y + 3z & xz + 2x + 4z & xy + 3x + 4y \end{vmatrix}$$

$$\mathbf{i} \left( \frac{\partial}{\partial y}(xy + 3x + 4y) - \frac{\partial}{\partial z}(xz + 2x + 4z) \right) - \mathbf{j} \left( \frac{\partial}{\partial x}(xy + 3x + 4y) - \frac{\partial}{\partial z}(yz + 2y + 3z) \right) + \mathbf{k} \left( \frac{\partial}{\partial x}(xz + 2x + 4z) - \frac{\partial}{\partial y}(yz + 2y + 3z) \right)$$

$$\mathbf{i}(x+4 - (x+1)) - \mathbf{j}(y+3 - (y+3)) + \mathbf{k}(z+2 - (z+2)) = 0\mathbf{i} - 0\mathbf{j} + 0\mathbf{k} = \langle 0, 0, 0 \rangle$$

Since  $\text{curl}(\mathbf{F}) = \langle 0, 0, 0 \rangle$ , answer is 0.