“QUIZ” for Lecture 24

NAME: (print!) Krithika Patrachari Section: 22

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q24FirstLast.pdf) ASAP BUT NO LATER THAN Dec. 4, 2020, 8:00pm

By using Stokes' Theorem, or otherwise, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$$F(x, y, z) = (yz + 2y + 3z)\mathbf{i} + (xz + 2x + 4z)\mathbf{j} + (xy + 3x + 4y)\mathbf{k} \quad ,$$

where C is the curve of intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=1$, oriented counterclockwise as viewed from above. Be sure to explain everything. + $r=1$

$z = 1-x-y$ $\quad$ $x^2+y^2=1$

$g(x,y) = 1-x-y$ $\quad$ $x=\sqrt{1-y^2}$ $\quad$ $y=\sqrt{1-x^2}$

$r=1$

$x = \cos\theta \quad y = \sin\theta \quad z = 0$

$r(t) = \cos\theta\, i + \sin\theta\, j + 0k \qquad 0 \le \theta \le 2\pi$

$F(x,y,z) = (0 + 2\sin\theta + 0)i + (0 + 2\sin\theta + 0)j + (\cos\theta\sin\theta + 3\cos\theta + 4\sin\theta)k$

$dr(t) = (2\cos\theta \ i + 2\cos\theta j + (-\sin\theta\cos\theta - 3\sin\theta + 4\cos\theta)k)\, d\theta$

$F \cdot dr = \langle 2\sin\theta, 2\sin\theta, \cos\theta\sin\theta + 3\cos\theta + 4\sin\theta \rangle \cdot \langle 2\cos\theta, 2\cos\theta, -\sin\theta\cos\theta - 3\sin\theta + 4\cos\theta \rangle$

$= (4\sin\theta\cos\theta + 4\sin\theta\cos\theta + (\cos\theta\sin\theta + 3\cos\theta + 4\sin\theta)(-\sin\theta\cos\theta - 3\sin\theta + 4\cos\theta))\, d\theta$

$\int_0^{2\pi} (4\sin\theta\cos\theta + 4\sin\theta\cos\theta + (\cos\theta\sin\theta + 3\cos\theta + 4\sin\theta)(-\sin\theta\cos\theta - 3\sin\theta + 4\cos\theta))\, d\theta$

$-4\cos\theta\sin\theta - 4\cos\theta\sin\theta + (-\sin\theta\cos\theta + 3\sin\theta - 4\cos\theta)(\cos\theta\sin\theta + 3\cos\theta + 4\sin\theta) \Big|_0^{2\pi}$

$(-0-0+(0+0-4)(0+3+0)) - (-0-0+(0+0-4)(0+3+0))$

$= -12 + 12 = 0$