"QUIZ" for Lecture 24

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q24FirstLast.pdf) ASAP BUT NO LATER THAN Dec. 4, 2020, 8:00pm

By using Stokes' Theorem, or otherwise, evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where

$$F(x,y,z) = (yz+2y+3z)\mathbf{i} + (xz+2x+4z)\mathbf{j} + (xy+3x+4y)\mathbf{k} \quad ,$$

where C is the curve if intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=1$, oriented counterclockwise as viewed from above. Be sure to explain everything.

Stokes' Theorem states that, for a vector field F:

$\int_C \vec{F}\cdot d\vec{r} = \int_S \text{curl}\,\vec{F}\cdot d\vec{S}$

If we take a look at the curve, we can actually see that it is a closed ellipse. So, if the vector field is conservative, its surface integral would be equal to 0:

F would be conservative if its $\text{curl}(F)=0$:

$$\text{curl}(\vec{F}) = \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz+2y+3z & xz+2x+4z & xy+3x+4y \end{vmatrix} =$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(xy+3x+4y) - \frac{\partial}{\partial z}(xz+2x+4z)\right) - \hat{j}\left(\frac{\partial}{\partial x}(xy+3x+4y) - \frac{\partial}{\partial z}(yz+2y+3z)\right) + \hat{k}\left(\frac{\partial}{\partial x}(xz+2x+4z) - \frac{\partial}{\partial y}(yz+2y+3z)\right) =$$

$$= \hat{i}(x+4-x-4) - \hat{j}(y+3-y-3) + \hat{k}(z+2-z-2) = \vec{0}$$

So, because the field is conservative, and the region is closed,

$\int_C \vec{F}\cdot d\vec{r} = \boxed{0}$