Quiz for Lecture 24

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section 22
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?1. By using Stokes' Theorem, or otherwise, evaluate $\int_C F\cdot dr$ where

$F(x,y,z) = (yz+2y+3z)i + (xz+2x+4z)j + (xy+3x+4y)k$.

where C is the curve of intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=1$, oriented counterclockwise as viewed from above. Be sure to explain everything.

① ~~$x=r\cos\theta$ $y=r\sin\theta$ $z=$~~

~~$=\cos\theta$~~ ~~$=\sin\theta$~~

$z=1-x-y$

$x=r\cos\theta \qquad y=r\sin\theta$

$\{(r,\theta) \mid 0\le r\le 1, \quad 0\le\theta\le 2\pi\}$

$\text{curl}\, F = (x+4-x-4)i - j(y+3-y-3) + k(z+2-z-2)$

$= \langle 0,0,0\rangle$

$\therefore F$ conservative

$$\begin{matrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ P & Q & R \end{matrix}$$

~~$\int_C F\cdot dr = \iint_S \text{curl}\, F\cdot ds$~~

$\therefore \int_C F\cdot dr = \int_S \text{curl} F\cdot ds$

$i: (R_y - Q_z)$

$j: (R_x - P_z)$

$k: (Q_x - P_y)$

Ans: 0.