

"QUIZ" for Lecture 23

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Section: 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalcz@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = (3x^2y^3z^3 + yz) \mathbf{i} + (3x^3y^2z^3 + xz) \mathbf{j} + (3x^3y^3z^2 + xy) \mathbf{k}$$

1. $\nabla F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3z^3+yz & 3x^3y^2z^3+xz & 3x^3y^3z^2+xy \end{vmatrix}$

$$= \mathbf{i}(d_y(3x^3y^3z^2+xy) - d_z(3x^3y^2z^3+xz)) - \mathbf{j}(d_x(3x^3y^3z^2+xy) - d_z(3x^2y^3z^3+yz)) + \mathbf{k}(d_x(3x^3y^2z^3+xz) - d_y(3x^2y^3z^3+yz))$$

$$= \mathbf{i}((9x^3y^3z^2+x) - (9x^3y^2z^3+x)) - \mathbf{j}((9x^3y^3z^2+xy) - (9x^2y^3z^3+yz)) + \mathbf{k}((9x^2y^2z^3+z) - (9x^2y^3z^3+z))$$

$$= 0\mathbf{i} - 0\mathbf{j} + 0\mathbf{k} = 0$$

the vector field F is conservative.

$$f_x = 3x^2y^3z^3 + yz$$

$$f = \int (3x^2y^3z^3 + yz) dx = x^3y^3z^3 + xyz + g(y, z)$$

$$f_y = 3x^3y^2z^3 + xz$$

$$3x^3y^2z^3 + xz + g_y = 3x^3y^2z^3 + xz$$

$$g_y = 0, \quad g(y, z) = h(z)$$

$$f = x^3y^3z^3 + xyz + h(z)$$

$$f_z = 3x^3y^3z^2 + xy$$

$$3x^3y^3z^2 + xy + h'(z) = 3x^3y^3z^2 + xy$$

$$h'(z) = 0$$

$$f = x^3y^3z^3 + xyz$$

2. Evaluate

$$\int_C 5y dx + 10x dy,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}.$$

Handwritten solution for the line integral problem:

$$2. \quad P = 5y \quad Q = 10x$$
$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial}{\partial x}(10x) - \frac{\partial}{\partial y}(5y) = 5$$
$$\iint_D 5 dA$$
$$= \int_0^1 \int_0^1 5 dx dy$$
$$\int_0^1 5 dx = 5x \Big|_0^1 = 5 - 0 = 5$$
$$\int_0^1 5 dy = 5y \Big|_0^1 = 5 - 0 = 5$$

Ans. 5