Vash Khangura "Quiz" for Lecture 23 Section 24

1.) Determine whether or not the vector field is conservative. If it is find a function f such that $F = \nabla f$

$$F(x,y,z) = (3x^2y^3z^3 + yz)i + (3x^3y^2z^3 + xz)j + (3x^3y^3z^2 + xy)k$$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 3x^2y^3z^3 + yz & 3x^3y^2z^3 + xz & 3x^3y^3z^2 + xy \end{vmatrix}$$

$$= [(9x^3y^2z^2 + x) - (9x^3y^2z^2 + x)]i + [(9x^2y^3z^2 + y) - (9x^2y^3z^2 + x)]j$$
$$+ [(9x^2y^2z^3 + z) - (9x^2y^2z^3 + z)]k = \langle 0,0,0 \rangle$$

B/c curl $F = 0$, F is a conservative vector field

$f_x = 3x^2y^3z^3 + yz \rightarrow f = x^3y^3z^3 + xyz$

$f_y = 3x^3y^2z^3 + xz \rightarrow f = x^3y^3z^3 + xyz$

$f_z = 3x^3y^3z^2 + xy \rightarrow f = x^3y^3z^3 + xyz$

Therefore, $f(x,y,z) = x^3y^3z^3 + xyz$

2.) Evaluate where C is the closed curve consisting of the boundary of the rectangle $\{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 1\}$

$\int_C 5y\,dx + 10x\,dy$ $\quad P = 5y \quad Q = 10x \quad \frac{dQ}{dx} - \frac{dP}{dy} = 10 - 5 = 5$

$$\int_0^1\int_0^1 5\,dxdy = \int_0^1 5\,dy = 5$$