

"QUIZ" for Lecture 23

Wenhao Li

NAME: (print!) _____ Section: All

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = (3x^2y^3z^3 + yz)\mathbf{i} + (3x^3y^2z^3 + xz)\mathbf{j} + (3x^3y^3z^2 + xy)\mathbf{k}$$

i	j	k
d/dx	d/dy	d/dz
$3x^2y^3z^3+yz$	$3x^3y^2z^3+xz$	$3x^3y^3z^2+xy$

$$i(d/dy(3x^3y^3z^2+xy)-d/dz(3x^3y^2z^3+xz)) - j(d/dx(3x^3y^3z^2+xy)-d/dz(3x^2y^3z^3+yz)) + k(d/dx(3x^3y^2z^3+xz) - d/dy(3x^2y^3z^3+yz)) = 0$$

$$f_x = 3x^2y^3z^3 + yz$$

$$f = x^3y^3z^3 + xyz + g(y, z)$$

$$f_y = 3x^3y^2z^3 + xz$$

$$3x^3y^2z^3 + xz + g_y = 3x^3y^2z^3 + xz$$

$$g_y = 0 \quad g(y, z) = h(z)$$

$$f = x^3y^3z^3 + xyz + h(z)$$

$$f_z = 3x^3y^3z^2 + xy$$

$$3x^3y^3z^2 + xy + h'(z) = 3x^3y^3z^2 + xy$$

$$h'(z) = 0$$

$$h(z) = 0$$

$$f = x^3y^3z^3 + xyz$$

2. Evaluate

$$\int_C 5y \, dx + 10x \, dy$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x, y) \mid 0 \leq x \leq 1, \quad 0 \leq y \leq 1\}.$$

$$P = 5y \quad Q = 10x$$

$$dQ/dx - dP/dy = 10 - 5 = 5$$

$$\int \int 5 \, dA$$

$$\int_0^1 \int_0^1 5 \, dx \, dy$$

$$= 5$$