"QUIZ" for Lecture 23

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = (3x^2y^3z^3 + yz)\,\mathbf{i} + (3x^3y^2z^3 + xz)\,\mathbf{j} + (3x^3y^3z^2 + xy)\,\mathbf{k}$$

$P_y = 9x^2y^2z^3 + z$ $\quad P_z = 9x^2y^3z^2 + y$ $\quad Q_z = 9x^3y^2z^2 + x$

$Q_x = 9x^2y^2z^3 + z$ $\quad R_x = 9x^2y^3z^2 + y$ $\quad R_y = 9x^3y^2z^2 + x$

Conservative because $P_y = Q_x$, $P_z = R_x$, $Q_z = R_y$

$\frac{\partial f}{\partial x} = 3x^2y^3z^3 + yz$ $\quad f = x^3y^3z^3 + xyz + g(y,z)$

$\frac{\partial f}{\partial y} = 3x^3y^2z^3 + xz + g'(y,z) = 3x^3y^2z^3 + xz$ $\quad g'(\cancel{y},z) = 0$

$f = x^3y^3z^3 + xyz + S(z)$ $\quad \frac{\partial f}{\partial z} = 3x^3y^3z^2 + xy + S'(z)$ $\quad S'(z) = 0$

$\boxed{F = \nabla f = x^3y^3z^3 + xyz}$

2. Evalute

$$\int_C 5y\,dx + 10x\,dy \quad ,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{\,(x, y)\,|\,0 \le x \le 1 \quad , \quad 0 \le y \le 1\,\}.$$

$\int_C 5y\,dx + 10x\,dy = \iint_C (10 - 5)\,dA = \iint_C 5\,dA$

$= \int_0^1\int_0^1 5\,dx\,dy \Rightarrow \int_0^1 5\,dx = 5$

$\int_0^1 5\,dy = \boxed{5}$