NAME: (print!) Yeram Sarah Jung   Section: 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = (3x^2 y^3 z^3 + yz)\,\mathbf{i} + (3x^3 y^2 z^3 + xz)\,\mathbf{j} + (3x^3 y^3 z^2 + xy)\,\mathbf{k}$$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 3x^2y^3z^3+yz & 3x^3y^2z^3+xz & 3x^3y^3z^2+xy \end{vmatrix}$$

$$= i\left((9x^3y^2z^2 + x) - (9x^3y^2z^2 + x)\right) = 0$$
$$- j\left((9x^2y^3z^2 + y) - (9x^2y^3z^2 + y)\right) = 0 \qquad 0i - 0j + 0k = 0 \quad \leftarrow \text{conservative}$$
$$+ k\left((9x^2y^2z^3 + z) - (9x^2y^2z^3 + z)\right) = 0$$

$$f_x = 3x^2y^3z^3 + yz$$
$$f = \int (3x^2y^3z^3 + yz)\,dx = x^3y^3z^3 + xyz + g(y,z)$$

$$\boxed{f = x^3y^3z^3 + xyz}$$

$$f_y = 3x^3y^2z^3 + xz = 3x^3y^2z^3 + xz + g(y) \qquad g(y) = 0$$
$$f_z = 3x^3y^3z^2 + xy = 3x^3y^3z^2 + xy + h(z) \qquad h(z) = 0$$

2. Evalute
$$\int_C 5y\,dx + 10x\,dy \quad,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x, y)\,|\,0 \le x \le 1 \quad,\quad 0 \le y \le 1\}.$$

$$\frac{d}{dy}(5y) = 5 \qquad \frac{d}{dx}(10x) = 10$$

$$10 - 5 = 5$$

$$\int_0^1 \int_0^y 5\,dx\,dy = 5x\Big|_0^y = 5y$$

$$\int 5y\,dy = \frac{5}{2}y^2\Big|_0^1 = \boxed{\frac{5}{2}}$$