

“QUIZ” for Lecture 23

NAME: (print!) SAI EMBAR Section: 83

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = (3x^2y^3z^3 + yz)\mathbf{i} + (3x^3y^2z^3 + xz)\mathbf{j} + (3x^3y^3z^2 + xy)\mathbf{k}$$

$$\frac{\partial P}{\partial y} = 9x^2y^2z^3 + z, \quad \frac{\partial Q}{\partial x} = 9x^2y^2z^3 + z$$

$$\frac{\partial P}{\partial z} = 9x^2y^2z^2 + y, \quad \frac{\partial R}{\partial x} = 9x^2y^2z^2 + y,$$

$$\frac{\partial Q}{\partial z} = 9x^3y^2z^2 + x, \quad \frac{\partial R}{\partial y} = 9x^3y^2z^2 + x$$

$$(3x^2y^3z^3 + yz)dx = x^3y^3z^3 + xy^2z + g(y, z)$$

$$3x^3y^2z^3 + xz + \frac{\partial g}{\partial y} = 3x^3y^2z^3 + xy^2z,$$

$$\frac{\partial g}{\partial y} = 0$$

$$3x^3y^2z^3 + xy^2z + g(z) = 3x^3y^2z^3 + xy^2z + C$$

$$\boxed{f(x, y, z) = x^3y^2z^3 + xy^2z + C}$$

2. Evaluate

$$\int_C 5y \, dx + 10x \, dy ,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}.$$

$$P = 5y, \quad Q = 10x.$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 10 - 5 = 5$$

$$\iint_D 5 \, dA = 5 \iint_D 1 \, dA = 5 \text{ Area}(D)$$

$$1, 1 = 1 \quad \text{so} \quad \text{Answer is } \boxed{5}.$$