"QUIZ" for Lecture 23

NAME: (print!) _SAI EMBAR_ Section: _83_

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = (3x^2y^3z^3 + yz)\,\mathbf{i} + (3x^3y^2z^3 + xz)\,\mathbf{j} + (3x^3y^3z^2 + xy)\,\mathbf{k}$$

$\frac{\partial P}{\partial y} = 9x^2y^2z^3 + z$, $\frac{\partial Q}{\partial x} = 9x^2y^2z^3 + z$

$\frac{\partial P}{\partial z} = 9x^2y^3z^2 + y$, $\frac{\partial R}{\partial x} = 9x^2y^3z^2 + y$)

$\frac{\partial Q}{\partial z} = 9x^3y^2z^2 + x$, $\frac{\partial R}{\partial y} = 9x^3y^2z^2 + x$

$\int (3x^2y^3z^3 + yz)\,dx = x^3y^3z^3 + xyz + g(y,z)$

$3x^3y^2z^3 + xz + \frac{\partial g}{\partial y} = 3x^3y^2z^3 + xz$,

$\frac{\partial g}{\partial y} = 0$

$3x^3y^3z^2 + xy + h'(z) = 3x^3y^3z^2 + xy$

$\boxed{f(x,y,z) = x^3y^3z^3 + xyz + C}$

2. Evalute
$$\int_C 5y\,dx + 10x\,dy \quad,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x,y)\,|\,0 \le x \le 1 \quad, \quad 0 \le y \le 1\}.$$

$P = 5y, \quad Q = 10x.$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 10 - 5 = 5$

$\iint_D 5\,dA = 5 \iint_D 1\,dA = 5\,\text{Area}(D)$

$1 \cdot 1 = 1 \qquad\qquad So \qquad \text{Answer is } \boxed{5}.$