

"QUIZ" for Lecture 23

NAME: (print!) \_\_\_\_\_ Section: \_\_\_\_\_

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function  $f$  such that  $\mathbf{F} = \nabla f$ .

$$\mathbf{F}(x, y, z) = (3x^2y^3z^3 + yz)\mathbf{i} + (3x^3y^2z^3 + xz)\mathbf{j} + (3x^3y^3z^2 + xy)\mathbf{k}$$

$$\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P(x, y, z) & Q(x, y, z) & R(x, y, z) \end{vmatrix} \Rightarrow \text{curl}(\mathbf{F}) = \langle 0, 0, 0 \rangle$$

$$\mathbf{F} = \langle f_x, f_y, f_z \rangle$$

$$f = \int f_x dx = x^3y^3z^3 + xyz$$

$$f(x, y, z) = x^3y^3z^3 + xyz$$

$$f_y = 3x^3y^2z^3 + xz + g_1, \quad g_1 = 0$$

$$f = x^3y^3 + z^3 + xyz + h(z)$$

$$f_z = 3x^3y^3z^2 + xy + h_z, \quad h_z = 0$$

2. Evaluate

$$\int_C 5y dx + 10x dy, \quad \text{where } C \text{ is the closed curve consisting of the boundary of the rectangle}$$

where  $C$  is the closed curve consisting of the boundary of the rectangle

$$\{(x, y) \mid 0 \leq x \leq 1, \quad 0 \leq y \leq 1\}.$$

Greene's Theorem:

$$\int_C P(x, y) dx + Q(x, y) dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$P(x, y) = 5y, \quad Q(x, y) = 10x$$

$$P_y = 5, \quad Q_x = 10$$

$$\int_0^1 \int_0^1 5 dx dy = 5 \left( \int_0^1 dx \right) \left( \int_0^1 dy \right) = \boxed{5}$$