

$$1. F(x, y, z) = (3x^2y^3z^3 + yz)i + (3x^3y^2z^3 + xz)j + (3x^3y^3z^2 + xy)k$$

$$\nabla F = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3z^3 + yz & 3x^3y^2z^3 + xz & 3x^3y^3z^2 + xy \end{vmatrix}$$

$$= i \left(\frac{\partial}{\partial y} (3x^3y^3z^2 + xy) - \frac{\partial}{\partial z} (3x^3y^2z^3 + xz) \right) - j \left(\frac{\partial}{\partial x} (3x^3y^3z^2 + xy) - \frac{\partial}{\partial z} (3x^2y^3z^3 + yz) \right)$$

$$+ k \left(\frac{\partial}{\partial x} (3x^3y^2z^3 + xz) - \frac{\partial}{\partial y} (3x^2y^3z^2 + yz) \right)$$

$$= i(9x^3y^2z^2 + x - 9x^3y^2z^2 - x) - j(9x^2y^3z^2 + y - 9x^2y^3z^2 - y)$$

$$+ k(9x^2y^2z^3 + z - 9x^2y^2z^3 - z)$$

$$= 0i - 0j + 0k = 0$$

$$f_x = 3x^2y^3z^3 + yz$$

$$f = \int (3x^2y^3z^3 + yz) dx = x^3y^3z^3 + xyz + g(y, z)$$

$$f_y = 3x^3y^2z^3 + xz$$

$$g(y, z) = 0, h(z) = 0$$

$$f = x^3y^3z^3 + xyz$$

$$2. \int_C 5y dx + 10x dy$$

$$P = 5y, Q = 10x$$

$$\frac{dQ}{dx} - \frac{dP}{dy} = 5y - 10x$$

$$\int_0^1 \int_0^1 (5y - 10x) dy dx$$

$$= -\frac{5}{2}$$

