

Quiz 23

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Section: 23

1. Determine whether or not the vector field is conservative

If it is, find a function f such that $F = \nabla f$

$$F(x, y, z) = (3x^2y^3z^3 + yz)\mathbf{i} + (3x^3y^2z^3 + xz)\mathbf{j} + (3x^3y^3z^2 + xy)\mathbf{k}.$$

$$\text{curl } F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3z^3 + yz & 3x^3y^2z^3 + xz & 3x^3y^3z^2 + xy \end{vmatrix}$$

$$= \mathbf{i}(9x^3y^2z^2 + x - 9x^3y^2z^2 - x) - \mathbf{j}(9x^2y^3z^2 + y - 9x^2y^3z^2 - y) + \mathbf{k}(9x^2y^2z^3 + z - 9x^2y^2z^3 - z).$$

$$= 0.$$

$\therefore F$ is conservative.

$$f_x = 3x^2y^3z^3 + yz \quad f = x^3y^3z^3 + xyz + g(y, z).$$

$$f_y = 3x^3y^2z^3 + xz \quad f = x^3y^3z^3 + xyz + h(z).$$

$$f_z = 3x^3y^3z^2 + xy \quad f = x^3y^3z^3 + xyz.$$

$$\therefore f = x^3y^3z^3 + xyz.$$



2. Evaluate

$$\int_C 5y dx + 10x dy$$

where C is the closed curve consisting of the boundary of the rectangle.

$$\{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$$

$$P = 5y \quad Q = 10x.$$

$$\int_0^1 \int_0^1 (0 - 5) dx dy$$

$$= 5 \times 1 \times 1$$

$$= 5.$$

