"QUIZ" for Lecture 23

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x,y,z) = (3x^2y^3z^3 + yz)\,\mathbf{i} + (3x^3y^2z^3 + xz)\,\mathbf{j} + (3x^3y^3z^2 + xy)\,\mathbf{k}$$

$\frac{\partial F_1}{\partial y} = 9y^2x^2z^3+z = \frac{\partial F_2}{\partial x} = 9x^2y^2z^3+z$

$\frac{\partial F_1}{\partial z} = 9z^2x^2y^3+y = \frac{\partial F_3}{\partial x} = 9x^2y^3z^2+y$

$\frac{\partial F_2}{\partial z} = 9z^2x^3y^2+x = \frac{\partial F_3}{\partial y} = 9y^2x^3z^2+x$

$\therefore$ the vector field is conservative.

$f(x,y,z) = \int 3x^2y^3z^3 + yz \, dx = x^3y^3z^3 + xyz + g(y,z)$

$\longrightarrow \frac{\partial}{\partial y}\left(x^3y^3z^3 + xyz + g(y,z)\right)$

$= 3y^2x^3z^3 + xz + g_y(y,z) = 3y^2x^3z^3 + xz$

$\longrightarrow \therefore g_y(y,z) = 0$

$\therefore g(y,z) = c$

potential function at $c=0$: $f(x,y,z) = x^3y^3z^3 + xyz$

2. Evalute

$$\int_C 5y\,dx + 10x\,dy \quad , \quad = \quad \iint 5 + 10 \, dA$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{\,(x,y)\,|\,0 \le x \le 1 \quad , \quad 0 \le y \le 1\,\}.$$

$\int_0^1 \int_0^1 15\,dx\,dy \longrightarrow \int_0^1 \left[\int_0^1 15\,dx\right] dy = \int_0^1 \left[15x\Big|_0^1\right] dy =$

$= \int_0^1 15\,dy = \boxed{15}$