"QUIZ" for Lecture 23

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x,y,z) = (3x^2y^3z^3 + yz)\,\mathbf{i} + (3x^3y^2z^3 + xz)\,\mathbf{j} + (3x^3y^3z^2 + xy)\,\mathbf{k}$$

The field is conservative, if $\operatorname{curl}(F) = \nabla \times F = \vec{0}$:

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3z^3+yz & 3x^3y^2z^3+xz & 3x^3y^3z^2+xy \end{vmatrix} = \hat{i}\left(\frac{\partial}{\partial y}(3x^3y^3z^2+xy) - \frac{\partial}{\partial z}(3x^3y^2z^3+xz)\right) - \hat{j}\left(\frac{\partial}{\partial x}(3x^3y^3z^2+xy) - \frac{\partial}{\partial z}(3x^2y^3z^3+yz)\right) + \hat{k}\left(\frac{\partial}{\partial x}(3x^3y^2z^3+xz) - \frac{\partial}{\partial y}(3x^2y^3z^3+yz)\right) =$$

$$= \hat{i}\,(9x^3y^2z^2+x - 9x^3y^2z^2 - x) - \hat{j}\,(9x^2y^3z^2+y - 9x^2y^3z^2 - y) + \hat{k}\,(9x^2y^2z^3+z - 9x^2y^2z^3 - z) = \vec{0} \quad \checkmark$$

The field is conservative, so, we can find a potential function f. Because $F[1] = f_x$, we can integrate $F[1]$ w.r.t. x:

$\int 3x^2y^3z^3 + yz\,dx = x^3y^3z^3 + xyz + g(y,z)$, where $g(y,z)$ is a function of only y and z.

Take the derivative of the result w.r.t. y and set it equal to $F[2]$ to find $g(y,z)$:

$3x^3y^2z^3 + xz + g_y = 3x^3y^2z^3 + xz \rightarrow g_y = 0 \rightarrow g(y,z) = 0 \Rightarrow f = x^3y^3z^3 + xyz + h(z)$

Take the derivative of the result w.r.t. z and set it equal to $F[3]$ to find $h(z)$:

$3x^3y^3z^2 + xy + h_z = 3x^3y^3z^2 + xy \Rightarrow h_z = 0 \Rightarrow h(z) = 0 \Rightarrow \boxed{f = x^3y^3z^3 + xyz}$

2. Evalute

$$\int_C 5y\,dx + 10x\,dy \quad ,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x,y)\,|\,0 \le x \le 1 \quad , \quad 0 \le y \le 1\}.$$

If we have functions $P(x,y)$ and $Q(x,y)$ in the line integral $\int_C P\,dx + Q\,dy$, we can use Green's Theorem to rewrite it as $\iint_D Q_x - P_y\,dA$.

So first, find the partial derivatives needed:

$Q = 10x \Rightarrow Q_x = 10 \qquad P = 5y \Rightarrow P_y = 5$

Our new integral, with the boundaries listed above, is:

$\int_0^1\int_0^1 10 - 5\,dy\,dx = \int_0^1\int_0^1 5\,dy\,dx = 5\cdot(1\cdot 1) = \boxed{5}$