

"QUIZ" for Lecture 23

NAME: (print!) Gillian Mulvey Section: _____

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $F = \nabla f$.

$$F(x, y, z) = (3x^2y^3z^3 + yz)\mathbf{i} + (3x^3y^2z^3 + xz)\mathbf{j} + (3x^3y^3z^2 + xy)\mathbf{k}$$

$$F \times \nabla = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3z^3 + yz & 3x^3y^2z^3 + xz & 3x^3y^3z^2 + xy \end{vmatrix} = (9x^3y^2z^2 + x - 9x^3y^2z^2 + x)\mathbf{i} - (9x^2y^3z^2 + y - 9x^2y^3z^2 + y)\mathbf{j} + (9x^2y^2z^3 + z - 9x^2y^2z^3 + z)\mathbf{k}$$

$$\frac{df}{dx} = 3x^2y^3z^3 + yz \quad \frac{df}{dy} = 3x^3y^2z^3 + xz \quad \frac{df}{dz} = 3x^3y^3z^2 + xy$$

$$f = x^3y^3z^3 + xyz + g(y, z) \quad f = x^3y^3z^3 + xyz \quad f = x^3y^3z^3 + xyz$$

$$x^3y^3z^3 + xyz = x^3y^3z^2 + xyz + g(y, z)$$

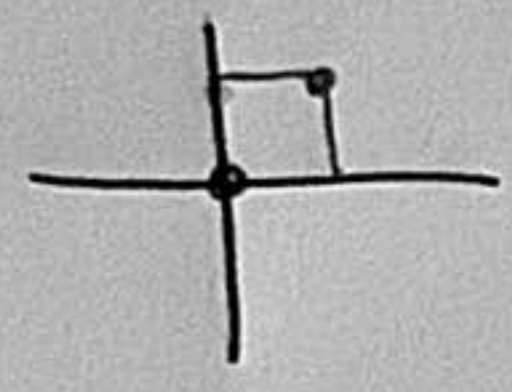
$$g(y, z) = 0$$

$$f = 2x^3y^3z^3 + 2xyz$$

2. Evaluate

$$\int_C 5y dx + 10x dy$$

where C is the closed curve consisting of the boundary of the rectangle



$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_C P dx + Q dy$$

$\{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

$$\iint_D 10 - 5 dA$$

$$\int_0^1 \int_0^1 5 dy dx = \int_0^1 5y \Big|_0^1 dx = \int_0^1 5 dx = 5x \Big|_0^1 = 5$$

or $5 \cdot \text{area} = 5 \cdot 1 \cdot 1 = 5$