

"QUIZ" for Lecture 23

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function  $f$  such that  $\mathbf{F} = \nabla f$ .

$$\mathbf{F}(x, y, z) = (3x^2y^3z^3 + yz)\mathbf{i} + (3x^3y^2z^3 + xz)\mathbf{j} + (3x^3y^3z^2 + xy)\mathbf{k}$$

$$\rightarrow \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ P(x,y,z) & Q(x,y,z) & R(x,y,z) \end{vmatrix} \text{ and the curl}(\mathbf{F}) = \langle 0, 0, 0 \rangle$$

$$\rightarrow f = \int f_x dx = x^3y^3z^3 + xyz$$

$$\rightarrow f_y = 3x^3y^2z^3 + xz + g_y \quad (g_y = 0)$$

$$\rightarrow f = x^3y^3z^3 + xyz + h(z)$$

$$\rightarrow f_z = 3x^3y^3z^2 + xy + h_z \quad (h_z = 0)$$

$$\rightarrow f(x, y, z) = x^3y^3z^3 + xyz$$

2. Evaluate

$$\int_C 5y dx + 10x dy$$

where  $C$  is the closed curve consisting of the boundary of the rectangle

$$\{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}.$$

$$\rightarrow \int P(x,y) dx + Q(x,y) dy = \int_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$\rightarrow P(x,y) = 5y$$

$$\rightarrow Q(x,y) = 10x$$

$$\rightarrow P_y = 5$$

$$\rightarrow Q_x = 10$$

$$\rightarrow \int_0^1 \int_0^1 5 dx dy = \boxed{5}$$