

"QUIZ" for Lecture 23

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function  $f$  such that  $\mathbf{F} = \nabla f$ .

$$\mathbf{F}(x, y, z) = (3x^2y^3z^3 + yz)\mathbf{i} + (3x^3y^2z^3 + xz)\mathbf{j} + (3x^3y^3z^2 + xy)\mathbf{k}$$

$$\begin{aligned} \nabla \times \mathbf{F} &= \langle 9x^3y^2z^2 + x - (9x^3y^2z^2 + x), 9x^2y^3z^2 + y - (9x^2y^3z^2 + y), 9x^2y^2z^3 + z - (9x^2y^2z^3 + z) \rangle \\ &= \langle 0, 0, 0 \rangle \end{aligned}$$

$$f = \int (3x^2y^3z^3 + yz) dx = x^3y^3z^3 + g(y, z)$$

$$3x^3y^2z^3 + g_y = 3x^3y^2z^3 + xz \quad g_y = xz$$

$$f = x^3y^3z^3 + xyz + h(z)$$

$$3x^3y^3z^2 + xy + h_z = 3x^3y^3z^2 + xy \quad h_z = 0$$

$$f = x^3y^3z^3 + xyz$$

2. Evaluate

$$\int_C 5y dx + 10x dy, \quad \text{where } C \text{ is the closed curve consisting of the boundary of the rectangle}$$

$\{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$ .

$$\int_0^1 \int_0^1 \left( \frac{\partial}{\partial x} (10x) - \frac{\partial}{\partial y} (5y) \right) dx dy$$

$$\int_0^1 \int_0^1 5 dx dy = 5$$