

"Quiz" for Lecture 23

Problem 1.

$$F(x,y,z) = (3x^2y^3z^3 + yz)\hat{i} + (3x^3y^2z^3 + xz)\hat{j} + (3x^3y^3z^2 + xy)\hat{k}$$

$\text{curl } \vec{F} = 0$ if vector field is conservative

$$\begin{aligned} \text{curl } \vec{F} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \begin{matrix} \uparrow \\ 3x^2y^3z^3 + yz \end{matrix} & \begin{matrix} \uparrow \\ 3x^3y^2z^3 + xz \end{matrix} & \begin{matrix} \uparrow \\ 3x^3y^3z^2 + xy \end{matrix} \end{vmatrix} \\ &= \left[(9x^3y^2z^2 + x) - (9x^3y^2z^2 + x) \right] \hat{i} \\ &\quad - \left[(9x^2y^3z^2 + y) - (9x^2y^3z^2 + y) \right] \hat{j} \\ &\quad + \left[(9x^2y^2z^3 + z) - (9x^2y^3z^3 + z) \right] \hat{k} \\ &= 0\hat{i} + 0\hat{j} + 0\hat{k} \Rightarrow F \text{ is a conservative vector field.} \end{aligned}$$

Find f such that $F = \nabla f$

$$\int P dx \Rightarrow \int (3x^2y^3z^3 + yz) dx = x^3y^3z^3 + xyz$$

$$\int Q dy \Rightarrow \int (3x^3y^2z^3 + xz) dy = x^3y^3z^3 + xyz$$

$$\int R dz \Rightarrow \int (3x^3y^3z^2 + xy) dz = x^3y^3z^3 + xyz$$

This implies ...

$$f = x^3y^3z^3 + xyz$$

Problem a.

$$\int_c 5y dx + 10x dy$$

$$\{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$$

points: (0, 0)

(1, 1)

$$r(t) = (1-t)(0, 0) + (1, 1)(t)$$

$$r(t) = (t, t)$$

$$\Rightarrow \int_c (5t)(1) + 10(t)(1) dt$$

$$= \int_0^1 15t dt = \frac{15}{2} t^2 \Big|_0^1 = \boxed{\frac{15}{2}}$$