"QUIZ" for Lecture 23

NAME: (print!) Ashwin Haridas **Section:** 22

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x,y,z) = \overset{P}{(3x^2y^3z^3 + yz)}\,\mathbf{i} + \overset{Q}{(3x^3y^2z^3 + xz)}\,\mathbf{j} + \overset{R}{(3x^3y^3z^2 + xy)}\,\mathbf{k}$$

$$\text{curl } F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix} = \hat{i}\left[(9x^3y^2z^2+x) - (9x^3y^2z^2+x)\right] - \hat{j}\left[9x^2y^3z^2+y - 9x^2y^3z^2+y\right]$$

$$= [0]$$

$\partial f/\partial x = (3x^2y^3z^3 + yz)$ $\quad \partial f/\partial y = (3x^3y^2z^3 + xz)$ $\quad \partial f/\partial z = (3x^3y^3z^2 + xy)$

$f = \int 3x^2y^3z^3 + yz\,dx = x^3y^3z^3 + xyz + g(y,z)$

$\dfrac{\partial f}{\partial y} = 3x^3y^2z^3 + xz + \dfrac{\partial g}{\partial y} = 3x^2y^2z^3 + xz \qquad \dfrac{\partial g}{\partial y} = 0$

$\int 0\,dz = 0 + h(z) \qquad \dfrac{\partial f}{\partial z} = 3x^3y^3z^2 + xy + \dfrac{\partial h}{\partial z} = 3x^3y^3z^2 + xy \qquad \dfrac{\partial h}{\partial z} = 0$

$$\boxed{f = x^3y^3z^2 + xyz}$$

2. Evalute

$$\int_C 5y\,dx + 10x\,dy \quad ,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{\,(x,y)\,|\,0 \le x \le 1 \quad , \quad 0 \le y \le 1\,\}.$$

$P = 5y \qquad Q = 10x$

$Q_x = 10 \qquad P_y = 5$

$Q_x - P_y = 5 \qquad \iint 5\,dA$

Since it is a constant, we can just do $5 \cdot 1 = \boxed{5}$

$\uparrow$ Area of rectangle.