"QUIZ" for Lecture 23

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x,y,z) = (3x^2y^3z^3 + yz)\,\mathbf{i} + (3x^3y^2z^3 + xz)\,\mathbf{j} + (3x^3y^3z^2 + xy)\,\mathbf{k}$$

$F(x,y,z) = P(x,y,z)i + Q(x,y,z)j + R(x,y,z)k$

$curl(F) = \langle 0,0,0 \rangle$

$F = \langle f_x, f_y, f_z \rangle$

$f(x,y,z) = x^3y^3z^3 + xyz$

$f = \int f_x\,dx = x^3y^3z^3 + xyz$

$f_y = 3x^3y^2z^3 + xz + g_y, \; g_y = 0$

$f = x^3y^3 + z^3 + xyz + h(z)$

$f_z = 3x^3y^3z^2 + xy + h_z, \; h_z = 0$

2. Evalute

$$\int_C 5y\,dx + 10x\,dy \quad ,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x,y)\,|\,0 \le x \le 1 \quad , \quad 0 \le y \le 1\}.$$

Greene's Theorem:

$\int_C P(x,y)dx + Q(x,y)dy = \iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy}\right)dA$

$P(x,y) = 5y \quad Q(x,y) = 10x \quad P_y = 5, \; Q_x = 10$

$\int_0^1\int_0^1 5\,dx\,dy = 5\left(\int_0^1 dx\right)\left(\int_0^1 dy\right) = \boxed{5}$