

"QUIZ" for Lecture 23

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: qXFirstLast.pdf) ASAP BUT NO LATER THAN Dec. 1, 2020, 8:00pm

1. Determine whether or not the vector field is conservative. If it is, find a function f such that $F = \nabla f$.

$$F(x, y, z) = (3x^2y^3z^3 + yz)\mathbf{i} + (3x^3y^2z^3 + xz)\mathbf{j} + (3x^3y^3z^2 + xy)\mathbf{k}$$

$$\text{curl}(F) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$$

$$f(x, y, z) = \int f_x dx = x^3y^3z^3 + xyz + h(y, z)$$

$$h(y, z) = 0 \quad g' = 0$$

$$f(x, y, z) = x^3y^3z^3 + xyz$$

$$\begin{aligned} &= \mathbf{i} (9x^3y^2z^4x - (9x^3y^2z^2 + x)) \\ &- \mathbf{j} (9x^2y^3z^2 + y - (9x^2y^3z^2 + y)) \\ &+ \mathbf{k} (9x^2y^2z^3 + z - (9x^2y^2z^3 + z)) \\ &= \mathbf{0}, \text{ conservative} \end{aligned}$$

2. Evaluate

$$\int_C 5y dx + 10x dy$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}.$$

$$= \int_0^1 \int_0^1 10 - 5 dy dx$$

$$= \int_0^1 \int_0^1 5$$

$$(\text{area})(\text{integrand}) = (1)(5) = \boxed{5}$$

$$\text{Area} = 1$$