Quiz for Lecture 23

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1. Determine whether or not the vector field is conservative. If it is, find a function f such that $F = \nabla f$.

$F(x, y, z) = (3x^2y^3z^3 + yz)i + (3x^3y^2z^3 + xz)j + (3x^3y^3z^2 + xy)k$

$P = (3x^2y^3z^3 + yz)$ $Q = (3x^3y^2z^3 + xz)$ $R = (3x^3y^3z^2 + xy)$.

$$\begin{matrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ P & Q & R \end{matrix}$$

$i: \frac{d}{dy}R - \frac{d}{dz}Q = (9x^3y^2z^2 + x) - (9x^3y^2z^2 + x)$
$= 0$

$j: \frac{d}{dx}R - \frac{d}{dz}P = (9x^2y^3z^2 + y) - (9x^2y^3z^2 + y)$
$= 0$

$k: \frac{d}{dx}Q - \frac{d}{dy}P = (9x^2y^2z^3 + z) - (9x^2y^2z^3 + z)$
$= 0$.

curl $F = \langle 0, 0, 0 \rangle$

conservative.

~~$f_x = \int F_x \, dx$~~

$f\_x = 3x^2y^3z^3 + yz$

$f = x^3y^3z^3 + xyz + g(y, z)$

$f\_y = 3x^3y^2z^3 + xz + g'(y, z) = 3x^3y^2z^3 + xz$

$g'(y, z) = 0$.

$g(y, z) = 0$.

$\boxed{f = x^3y^3z^3 + xyz}$

$f = x^3y^3z^3 + xyz + h(z)$

$f\_z = 3x^3y^3z^2 + xy + h'(z) = 3x^3y^3z^2 + xy$

$h'(z) = 0$ $h(z) = 0$.

2. Evaluate

$$\int_C 5y\,dx + 10x\,dy.$$

where C is the closed curve consisting of the boundary of the rectangle.

$\{(x,y) \mid 0 \leq x \leq 1,\quad 0 \leq y \leq 1\}.$

$P = 5y \qquad Q = 10x$

$\frac{d}{dx}Q - \frac{d}{dy}P = 10 - 5 = 5.$

$\int_0^1 \int_0^1 5\,dx\,dy = 5 \times 1 = 5.$

$\boxed{\text{Ans: } 5.}$