

"QUIZ" for Lecture 22

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q22FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field \mathbf{F} and oriented surface S .

$$\mathbf{F}(x, y, z) = \langle xy, yz, zx \rangle,$$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$ and has upward orientation.

The image shows a handwritten solution on lined paper. The work is as follows:

$$g = 1 - x^2 - y^2$$
$$P = xy, Q = yz, R = zx$$
$$\iint_S \mathbf{F} \cdot d\mathbf{s} = \iint_D -xy(-2x) - yz(-2y) + zx \, dA$$
$$= \iint_D (2x^2y + (2y^2 + x)z) \, dA$$
$$= \iint_D (2x^2y + (2y^2 + x)(1 - x^2 - y^2)) \, dA$$
$$= \iint_D (2x^2y + 2y^2 - 2x^2y^2 - 2y^4 + x - x^3 - xy^2) \, dA$$
$$\int_0^1 \int_0^1 (2x^2y + 2y^2 - 2x^2y^2 - 2y^4 + x - x^3 - xy^2) \, dx \, dy$$
$$= \int_0^1 \left(2y^4 + \frac{4y^2}{3} + \frac{y}{6} + \frac{1}{4} \right) dy$$
$$= \frac{17}{45}$$