"QUIZ" for Lecture 22

NAME: (print!) Shaun Goda **Section:** 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q22FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

Evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and oriented surface S.

$$\mathbf{F}(x, y, z) = \langle xy\,,\, yz\,,\, zx \rangle \quad ,$$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \le x \le 1\,,\ 0 \le y \le 1$ and has upward orientation.

$g = 1 - x^2 - y^2 \quad P = xy \quad Q = yz \quad R = zx$

$g_x = -2x$

$g_y = -2y$

$$\iint_S F\,ds = \int\int_D \Big(-(xy)(-2x) - (yz)(-2y) + (zx) \Big)\, dA$$

$$= \int\int_D \left(2x^2 y + 2y^2 z + z x \right) dA$$

$$= \int\int_D \left(2x^2 y + 2y(1 - x^2 - y^2) + (1 - x^2 - y^2)\, x \right) dA$$

$$= \int\int_D \left(2x^2 y + 2y - 2x^2 y - 2y^3 + x - x^3 - x y^2 \right) dA$$

$$= \int\int_D \left(2y - 3x^2 y - 2y^3 + x - x^3 \right) dA$$

$$= \int_0^1 \int_0^1 \left(2y - 3x^2 y - 2y^3 + x - x^3 \right) dx\, dy = \boxed{\frac{17}{12}}$$