

"QUIZ" for Lecture 22

NAME: (print!) \_\_\_\_\_ Section: \_\_\_\_\_

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q22FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

Evaluate the surface integral  $\int \int_S \mathbf{F} \cdot d\mathbf{S}$  for the given vector field  $\mathbf{F}$  and oriented surface  $S$ .

$$\mathbf{F}(x, y, z) = \langle xy, yz, zx \rangle,$$

and  $S$  is the part of the paraboloid  $z = 1 - x^2 - y^2$  that lies above the square  $0 \leq x \leq 1, 0 \leq y \leq 1$  and has upward orientation.

$$\begin{aligned} \mathbf{r} &= xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k} \\ \mathbf{r}_x &= y\mathbf{i} + z\mathbf{j} + z\mathbf{k} \\ \mathbf{r}_y &= x\mathbf{i} + z\mathbf{j} + zx\mathbf{k} \\ d\mathbf{S} &= \mathbf{r}_x \times \mathbf{r}_y \\ &= \langle yz^2x - z^2, zx - yz^2, yz - zyx \rangle \\ \int_S |d\mathbf{S}| &= \int_A \sqrt{1 + (2x)^2 + (2y)^2} \, dA \\ &= \int_0^1 r \, dr \int_0^{2\pi} d\theta \sqrt{4r^2 + 1} \\ &= \frac{\pi(5\sqrt{5} - 1)}{6} \end{aligned}$$