Quiz for ~~ZZ~~. Lecture 22.

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Section: 23.

Evaluate the surface integral $\iint_S F \cdot dS$ for the given vector field F and oriented surface S.

$$F(x,y,z) = \langle xy, yz, xz \rangle$$

and S is the part of the paraboloid $z = 1-x^2-y^2$ that lies above the square $0 \le x \le 1$, $0 \le y \le 1$ and has upward orientation

$$g = 1-x^2-y^2$$

$$P = xy,\ Q = yz,\ R = xz.$$

$$\iint_S F \cdot dS$$

$$= \iint_D 2x^2y + (2y^2 + x)z \, dA$$

$$= \iint_D 2x^2y + (x+2y^2)(1-x^2-y^2) \, dA$$

$$= \iint_D (2x^2y + x - x^3 - xy^2 + 2y^2 - 2x^2y^2 - 2y^4) \, dA.$$

$$= \int_0^1 \int_0^1 (2x^2y + x - x^3 - xy^2 + 2y^2 - 2x^2y^2 - 2y^4) \, dx \, dy$$

Inner Loop: $\left[\frac{2x^3y}{3} + \frac{1}{2}x^2 - \frac{1}{4}x^4 - \frac{1}{2}x^2y^2 + 2xy^2 - \frac{2}{3}x^3y^2 - 2xy^4\right]_0^1$

$$= -2y^4 + \frac{5}{6}y^2 + \frac{2}{3}y + \frac{1}{4}$$

Outer Loop: $\left[-\frac{2}{5}y^5 + \frac{5}{18}y^3 + \frac{1}{3}y^2 + \frac{1}{4}y\right]_0^1$

$$= \frac{93}{180}$$