"QUIZ" for Lecture 22

NAME: (print!) Jennifer Gonzalez **Section:** 2

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q22FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

Evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and oriented surface S.

$$\mathbf{F}(x, y, z) = \langle xy\,,\, yz\,,\, zx \rangle \quad ,$$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \le x \le 1\,,\, 0 \le y \le 1$ and has upward orientation.

$g = (1-x^2-y^2)$ $\quad P = xy\,,\quad Q = yz\,,\quad R = zx$

$$\iint_D \left(-xy\,(2x) - yz\,(2y)\right) dA$$

$$= \iint_D \left(-2x^2y - y^2(1-x^2-y^2)^2\right) dA$$

$$= \iint_D \left(-2x^2y - y^2 - (1 - x^2 - y^2 - x^2 + x^4 + x^2y^2 - y^2 + x^2y^2 + y^4)\right) dA$$

$$= \iint_D \left(-2x^2y - y^2 - 1 + x^2 + y^2 + x^2 - x^4 - x^2y^2 + y^2 + x^2y^2 + y^4\right) dA$$

$$= \iint_D -2x^2y + 2y^2 + y^4 + 2x^2 - x^4 - 1 \; dA$$

$$= \int_0^1 \int_0^1 -2x^2y + 2y^2 + y^4 + 2x^2 - x^4 - 1 \; dx\,dy$$

$$= \int_0^1 \left(-2y\,\frac{x^3}{3} + 2y^2x + y^4x + 2\,\frac{x^3}{3} - \frac{x^5}{5} - x\right)\Big|_0^1 dy$$

$$= \int_0^1 \left(-\frac{2y}{3} + 2y^2 + y^4 + \frac{2}{3} - \frac{1}{5} - 1\right) dy = -\frac{2y^2}{6} + \frac{2}{3}y^3 + \frac{y^5}{5} + \frac{2y}{3} - \frac{y}{5} - y \Big|_0^1$$

$$= \frac{2}{6} + \frac{2}{3} + \frac{1}{5} + \frac{2}{3} - \frac{1}{5} - 1 = \frac{4}{3} - 1 = \boxed{\frac{1}{3}}$$