"QUIZ" for Lecture 22

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q22FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

Evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and oriented surface S.

$$\mathbf{F}(x, y, z) = \langle xy\,,\, yz\,,\, zx \rangle \quad ,$$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1\,,\, 0 \leq y \leq 1$ and has upward orientation.

The formula for the line integral of the vector field over the surface S with upward orientation is:

$\cdot \iint_S F \cdot dS = \iint_D \left(-P \frac{\partial g}{\partial x} - Q \frac{\partial g}{\partial y} + R\right) dA$, where $g(x,y) = \cancel{z}$

So first, find the partial derivatives of $g(x,y)$:

$\frac{\partial g}{\partial x} = -2x \quad \frac{\partial g}{\partial y} = -2y$

If $P = xy$, $Q = yz$, $R = zx$, and $z = 1 - x^2 - y^2$, plug them into the formula above:

$\iint_S F \cdot dS = \iint_D -xy(-2x) - yz(-2y) + zx \, dA =$

$= \iint_D 2x^2y + 2y^2(1-x^2-y^2) + (1-x^2-y^2)x \, dA =$

$= \iint_D 2x^2y + 2y^2 - 2x^2y^2 - 2y^4 + x - x^3 - xy^2 \, dA$

The region D, in this case, has bounds $0 \leq x \leq 1$ and $0 \leq y \leq 1$. So, our integral is:

$\int_0^1\int_0^1 2x^2y + 2y^2 - 2x^2y^2 - 2y^4 + x - x^3 - xy^2 \, dx\, dy =$

$= \int_0^1 \frac{2x^3y}{3} + 2y^2x - \frac{2x^3y^2}{3} - 2y^4x + \frac{x^2}{2} - \frac{x^4}{4} - \frac{x^2y^2}{2} \Big|_0^1 dy =$

$= \int_0^1 \frac{2y}{3} + 2y^2 - \frac{2y^2}{3} - 2y^4 + \frac{1}{2} - \frac{1}{4} - \frac{y^2}{2} \, dy =$

$$= \int_0^1 \frac{2y}{3} + 2y^2 - \frac{2y^2}{3} - 2y^4 + \frac{1}{4} - \frac{y^2}{2}\, dy =$$

$$= \frac{y^2}{3} + \frac{2y^3}{3} - \frac{2y^3}{9} - \frac{2y^5}{5} + \frac{y}{4} - \frac{y^3}{6}\Big|_0^1 =$$

$$= \frac{1}{3} + \frac{2}{3} - \frac{2}{9} - \frac{2}{5} + \frac{1}{4} - \frac{1}{6} = {}^{\backslash 5}\frac{7}{9} - {}^{\backslash 9}\frac{2}{5} + {}^{\backslash 3}\frac{1}{4} - {}^{\backslash 2}\frac{1}{6} = \left(\frac{35-18}{45}\right) + \left(\frac{3-2}{12}\right) =$$

$$= {}^{\backslash 12}\frac{17}{45} + {}^{\backslash 45}\frac{1}{12} = \frac{204+45}{540} = \frac{249}{540} = \boxed{\frac{83}{180}}$$