"QUIZ" for Lecture 22

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q22FirstLast.pdf) ASAP BUT NO LATER THAN Nov. 16, 8:00pm

Evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and oriented surface S.

$$\mathbf{F}(x, y, z) = \langle xy\,,\, yz\,,\, zx \rangle \quad ,$$

and S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the square $0 \le x \le 1, 0 \le y \le 1$ and has upward orientation.

$g = 1 - x^2 - y^2$

$$\iint_S F \cdot dS = \iint_D -P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \, dA$$

$$= \iint_D -xy(-2x) - yz(-2y) + zx \, dA = \iint_D 2x^2y + 2y^2z + zx \, dA$$

$$= \iint_D 2x^2y + (2y^2 - 2x^2y^2 - 2y^4) + (x - x^3 - xy^2) \, dA$$

$$= \int_0^1 \int_0^1 2x^2y + 2y^2 - 2x^2y^2 - 2y^4 + x - x^3 - xy^2 \, dy\,dx$$

$$= \int_0^1 x^2y^2 + \frac{2}{3}y^3 - \frac{2}{3}x^2y^3 - \frac{2}{5}y^5 + xy - \frac{x}{3}y^3 \Big|_0^1 \, dx$$

$$= \int_0^1 x^2 + \frac{2}{3} - \frac{2}{3}x^2 - \frac{2}{5} + x - x^3 - \frac{x}{3} \, dx$$

$$= \frac{x^3}{3} + \frac{2}{3}x - \frac{2}{9}x^3 - \frac{2}{5}x + \frac{x^2}{2} - \frac{x^4}{4} - \frac{x^2}{2} \Big|_0^1$$

$$= \frac{1}{3} + \frac{2}{3} - \frac{2}{9} - \frac{2}{5} + \frac{1}{2} - \frac{1}{4} - \frac{1}{6}$$

$$= \frac{83}{180}$$