1. $x_u = v$ $y_u = w$ $z_u = 1$

$x_v = u$ $y_v = 1$ $z_v = w$

$x_w = 1$ $y_w = u$ $z_w = v$

plug in $(u, v, w) = (2, 2, 2)$

$$\begin{matrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{matrix}$$

$2(-2) - 2(2) + 3$

$= -4 - 4 + 3 = -5$

2.

(i) $\begin{matrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2yz & +yz + \cos(x+y+z) \ldots\ldots \end{matrix}$

$= i\left(\frac{\partial}{\partial y}(x^3y + xy + \cos(x+y+z)) - \frac{\partial}{\partial z}(x^3z + xz + \cos(x+y+z))\right) - j \ldots + k \ldots$

$= 0i + 0j + 0k = (0, 0, 0)$

(ii)

$f = \int (3x^2yz + yz + \cos(x+y+z))dx = x^3yz + xyz + \sin(x+y+z) + g(y, z)$

$f_y = x^3z + xz + \cos(x+y+z)$

$x^3z + xz + \cos(x+y+z) + g_y = x^3z + xz + \cos(x+y+z)$

$g_y = 0.$ $h(z) = 0$

$f = x^3yz + xyz + \sin(x+y+z) + h(z)$

$f_z = x^3y + xy + \cos(x+y+z)$

$x^3y + xy + \cos(x+y+z)$ $+h'(z) = x^3y + xy + \cos(x+y+z)$

$h(z) = 0.$ $h(z) = 0$

$f = x^3yz + xyz + \sin(x+y+z)$

(iii) $r(0) = (0, 2, 0)$ $r(\pi) = (0, 0, 0)$

$$\int_C F \cdot dr = f(0, 0, 0) - f(0, 2, 0)$$

$$= -\sin 2.$$

3.

$1 \le x \le 2$
$0 \le y \le e^x + 1$

$y = e^x + 1$

$x = \ln(y-1)$

$(x,y) : 1 \le y \le e+1, \ 1 \le x \le 2$

$(x,y) \ e+1 \le y \le e^2+1, \ \ln(y-1) \le x \le 2$

$$\int_0^{e+1} \int_1^2 F(x,y)\,dx\,dy + \int_{e+1}^{e^2+1} \int_{\ln(y-1)}^{2} F(x,y)\,dx\,dy.$$

4.

$f = x+y+z \quad g = xyz - 1$

$\nabla f = \langle 1,1,1 \rangle$
$\nabla g = \langle yz, xz, xy \rangle$
$\langle 1,1,1 \rangle = L \langle yz, xz, xy \rangle$

$1 = Lyz \quad 1 = Lxz \quad 1 = Lxy$

$x = Lxyz \quad y = Lxyz \quad z = Lxyz$

$xyz = 1 \quad \cancel{x = y = z}$

$L^3 x^2 y^2 z^2 = 1$
$L^3 \times 1 \times 1 = 1$

$L^3 = 1$
$L = 1$

$x = 1 \quad y = 1 \quad z = 1$
$f = x+y+z = 1+1+1$
$= 3.$

5.

$(x,y,z) \ 0 \le z \le 1 \ 0 \le y \le z \ 0 \le x \le y.$

$$\int_0^1 \int_0^z \int_0^y 48xyz\,d\,x\,dy\,dz$$

$= 1$

6. $\{(r,\theta) \ 0 \le r \le 3, \ 0 \le \theta \le 2\pi\}.$

$$\int_0^{2\pi}\int_0^3 \frac{r^4}{243\pi} r\,dr\,d\theta$$

$= 1.$

how to get r, θ.

7. $\langle 0,0,0 \rangle + t\langle 1,1,1 \rangle = \langle t,t,t \rangle$

$r(t) = \langle t,t,t \rangle \qquad 0 \le t \le 1$
$r'(t) = \langle 1,1,1 \rangle.$

$|r'(t)| = \sqrt{3}$

$$\int_0^1 \frac{4(\sqrt{3}\,t)^3}{3} \cdot \sqrt{3}\,dt = 1.$$

8.

$\sqrt{\frac{y}{3}} \le x \le 1$

$0 \le y \le 3$

$x = \sqrt{\frac{y}{3}}$

$y = 3x^2$

$0 \le y \le 3x^2$

$0 \le x \le 1$

$\int_0^1 \int_0^{3x^2} e^{x^3} dy\, dx$

$= e-1$

9. $\{(\rho, \phi, \theta) \mid 0 \le \rho \le 1, \quad 0 \le \phi \le \pi, 0 \le \theta \le 2\pi\}$

$x^2+y^2+z^2 = \rho^2 \quad dv = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$\frac{5\pi}{4} \int_0^1 \int_0^{\pi} \int_0^{2\pi} \rho^4 \sin\phi\, d\theta\, d\phi\, d\rho$

$= \frac{5\pi}{4} \int_0^1 \rho^4 d\rho \cdot \int_0^{\pi} \sin\phi\, d\phi \cdot \int_0^{2\pi} d\theta.$

$= 1$

10. $\nabla \cdot F = \cancel{(} \frac{\partial}{\partial x}(\sin(xy)) + \frac{\partial}{\partial y}(\sin(yz)) + \frac{\partial}{\partial z}(\sin(xz)).$

$= y\cos(xy) + z\cos(yz) + x\cos(xz).$

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