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11/19/2020

Practice Midterm 2

1 $x = uv^2 + w \qquad y = uw^2 + v \qquad z = vw + u^2$

$$\begin{bmatrix} \cancel{\partial f/\partial x}\ \partial x/\partial u & \partial x/\partial v & \partial x/\partial w \\ \partial y/\partial u & \partial y/\partial v & \partial y/\partial w \\ \partial z/\partial u & \partial z/\partial v & \partial z/\partial w \end{bmatrix}$$

Plug in (1,1,1) into each to get

$$\begin{bmatrix} v^2 & 2vu & 1 \\ w^2 & 1 & 2wu \\ 2u & w & v \end{bmatrix}$$

$$\det \begin{bmatrix} 1^2 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix} = 3$$

2 $\vec{F}(x,y) = (x^2+y^2)\hat{i} + (2xy)\hat{j}$

$\frac{\partial f}{\partial x} = x^2+y^2 \qquad \frac{\partial F}{\partial y} = 2xy$

$f = \frac{x^3}{3} + xy^2 + g(y) \qquad f(x,y) = xy^2 + f(x)$

$F(x,y) = \frac{x^3}{3} + xy^2$

$\oint_C \vec{F} \cdot d\vec{r} = 0$

3 $\int_0^5 \int_0^{3x} F(x,y)\, dy\, dx$

$0 \leq x \leq 5 \qquad 0 \leq y \leq 3x$

Infinitely adding in y-direction (dy) first, then moving in x-direction (dx)

X-axis ranges from (0,0) to (5,0)
y ranges (0,0) to (0,3x)
y = 3x is the roof

4 $\quad Dg = D(x^2+y^2) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

$$D(x^2y) = \begin{bmatrix} 2xy \\ x^2 \end{bmatrix}$$

$$\begin{bmatrix} 2xy \\ x^2 \end{bmatrix} = \lambda \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$2xy = \lambda 2x \rightarrow 2y = \lambda 2 \rightarrow y = \lambda$

$x^2 = \lambda 2y \rightarrow x^2 = 2y^2$

$x^2 + y^2 = 1$

$\rightarrow 2y^2 + y^2 = 1 =$

$3y^2 = 1$

$\lambda = y = \pm\sqrt{1/3}$

$x = \pm\sqrt{2/3}$

5 $$\int_0^3\int_0^4\int_0^2 xyz \, dz \, dy \, dx$$

$$\left[\frac{xyz^2}{2}\right]_0^2 = 2xy$$

$$\int_0^3\int_0^4 2xy \quad dy \, dx$$

$$\left[\frac{2xy^2}{2}\right]_0^4$$

$$\left[\frac{16x^2}{2}\right]_0^3 = 72$$

6 $$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{(x^2+y^2)^2}{243\pi} \, dy \, dx$$

$$\frac{1}{243\pi}\int_0^{2\pi}\int_0^3 \frac{r^4}{\cancel{243\pi}} r\,dr\,d\theta \rightarrow \left(\int_0^{2\pi} d\theta\right)\left(\int_0^3 r^5 dr\right)$$

$$\left[\frac{r^6}{6}\right]_0^3 (2\pi)\left(\tfrac{1}{243\pi}\right)$$

$$= 1$$

7 $1-0=1$ $(0,0,0)+t(1,1,1)-(0,0,0)$

$=(t,t,t)$

$(0,1)$

$$\int_0^1 \cancel{4\sqrt{3}}\ \frac{4t^3}{3}\,dt =$$

$$\frac{4t^4}{4}\Big|_0^1 = 1$$

8

$$\int_0^3 \int_{x^3}^1 e^{3x}\,dx\,dy$$

$$\int_{x^3}^1 e^{3x}\,dx \rightarrow \frac{1}{3}\left(e^3 - e^{3x^3}\right)$$

$$\int_0^3 \left(\frac{1}{3}\right)\left(e^3 - e^{3x^3}\right)dy$$

$$\cancel{3}\left(e^3 - e^{3x^3}\right)$$

9 $$\int_0^1\int_0^\pi\int_0^{4\pi} \frac{4(x^2+y^2+z^2)}{4\pi}\, dV$$

$$\frac{4}{3}(3y^2+3z^2+16\pi^2)$$

$$\hookrightarrow \frac{4(3y^2+3z^2+16\pi^2)}{3}$$

$$\hookrightarrow \frac{4(\cancel{3\pi z^2+17\pi^3})}{3} \quad 17\pi^3+\pi$$

$$\frac{4}{3}(17\pi^3+\pi)$$

10 $$\nabla\cdot F = f_x + F_y + f_z$$

$$y\cos(xy) + z\cos(yz) + x\cos(xz)$$