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Section: ____ **E-Mail address:** ____________________

MATH 251 (4,6,7), Dr. Z. , Exam 2, Tue., Nov. 21, 2017, SEC 118

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

MAKE SURE TO PUT THE TYPE!
Types: Number, Function of *variable*(s), 2D vector of numbers, 3D vector of numbers, 2D vector of functions (aka 2D vector-field), 3D vector of functions (aka 3D vector field), equation of a plane, parametric equation of a line, equation of a line, equation of a surface, equation of a line, DNE (does not exist), abstract double-integral, abstract triple-integral.

1. (10 pts.)
Find the Jacobian of the transformation from (u, v, w)-space to (x, y, z)-space.

$$x = uv + w \quad , \quad y = uw + v \quad , \quad z = vw + u \quad ,$$

at the point ~~$(u, v, w) = (2, 2, 2)$~~.
$(u, v, w) = (1, 1, 1)$

The **type** of the answers is:

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ans.

$$\det \begin{pmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{pmatrix}$$

$x_u = v \qquad x_v = u \qquad x_w = 1$

$y_u = w \qquad y_v = 1 \qquad y_w = u$

$z_u = 1 \qquad z_v = w \qquad z_w = v$

$$\det \begin{pmatrix} v & u & 1 \\ w & 1 & u \\ 1 & w & v \end{pmatrix} = u^2 - 2uvw + v^2 + w^2 - 1$$

$$\mathrm{Jac}(1,1,1) = 1 - 2 + 1 + 1 - 1$$
$$= 3 - 2 - 1$$
$$= -2$$

$\mathrm{Jac}(1,1,1) = -2$

2. (10 points altogether)
(i) (3 points) Show that

$$\mathbf{F} = \langle \overset{2}{\cancel{3}}x^2yz+yz+\cos(x+y+z)\ ,\ x^3z+xz+\cos(x+y+z)\ ,\ x^3y+xy+\cos(x+y+z) \rangle \quad ,$$

is a conservative vector field.

(ii) (4 point) Find a function $f(x,y,z)$ such that $\mathbf{F} = \bigtriangledown f$.

(iii) (3 points) Find the line-integral $\int_C \mathbf{F}.d\mathbf{r}$ where C is the curve

$$\mathbf{r} = \langle \sin t, \cos t + 1, \sin \overset{t}{\cancel{2t}} \rangle \quad , \quad 0 \le t \le \pi \quad .$$

The **types** of the answer is: For (ii) For (iii)

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answers (ii) f(x,y,z)=

(iii)

$$\mathbf{F} = \langle \underset{P}{\overset{2}{\cancel{3}}x^2yz+yz+\cos(x+y+z)}\ ,\ \underset{Q}{x^3z+xz+\cos(x+y+z)}\ ,\ \underset{R}{x^3y+xy+\cos(x+y+z)} \rangle$$

Find $\nabla \times F$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \langle 0, -x^2y, x^2z \rangle$$

F is not conservative,
therefore problem must end
here.

3. (10 points)
Sketch the region of integration and change the order of integration.

$$\int_{1=x}^{\cancel{2}\;3=x} \int_{0=y}^{\cancel{e^x+1=y}\;e^x+2=y} F(x,y)\,dy\,dx$$

The **type** of the answer is:

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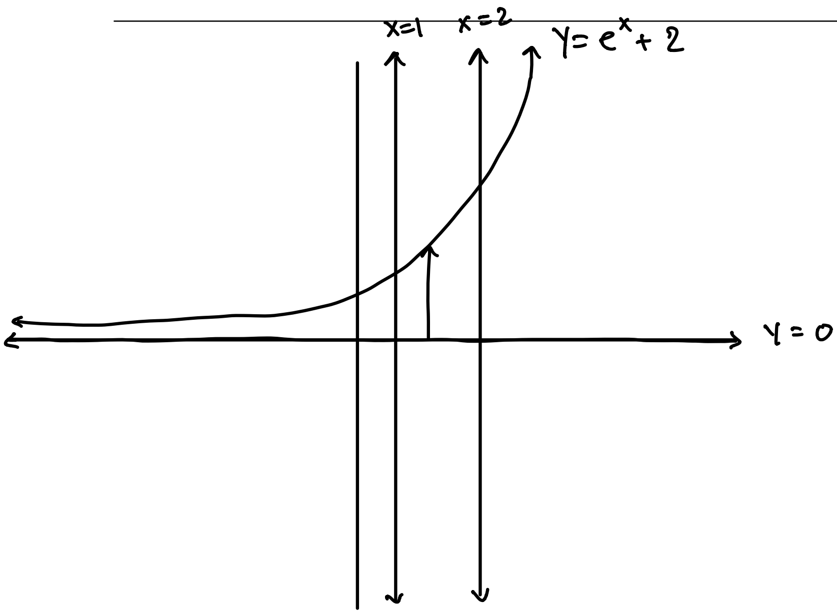
ans.



$$\int\int f(x,y)\,dx\,dy$$

$$\int_{e+2}^{e^2+2}\int_0^{\ln(y-2)} F(x,y)\,dx\,dy$$

$$\int_{e+1}^{e^2+1}\int_0^{\ln(y-1)} F(x,y)\,dx\,dy$$

4. (10 points) Use Largange multipliers (no credit for other methods) to find the smallest value that $x + y + z$ can be, given that $\cancel{xyz = 1}$.
$xyz = 9$

The **type** of the answer is:

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ans.

$f(x,y,z) = x+y+z$, $g(x,y,z) = xyz = 27$

$\nabla f = \langle 1,1,1 \rangle$ $\nabla g = \langle yz, xz, xy \rangle$

$\nabla f = \lambda \nabla g$

$\langle 1,1,1 \rangle = \lambda \langle yz, xz, xy \rangle$

$\frac{1}{\lambda} = yz$, $\frac{1}{\lambda} = xz$, $\frac{1}{\lambda} = xy$

$(xyz)^2 = \frac{1}{\lambda^3}$

$(yz)^2 = \frac{1}{x^2\lambda^3}$

$\frac{1}{\lambda^2} = \frac{1}{x^2\lambda^3}$

$\lambda^2 = x^2\lambda^3$

$x = \sqrt{\frac{1}{\lambda}}$

$x = \sqrt{\frac{1}{\lambda}}$

$y = \sqrt{\frac{1}{\lambda}}$

$z = \sqrt{\frac{1}{\lambda}}$

$xyz = 27$

$\frac{1}{\lambda}\sqrt{\frac{1}{\lambda}} = 27$

$\frac{1}{\lambda^3} = 729$

$\frac{1}{729} = \lambda^3$

$\sqrt[3]{\frac{1}{729}} = \lambda$

$\frac{1}{9} = \lambda$

$x = \sqrt{9} = 3$

$x, y, z = 3$

$x + y + z = 9$

$\boxed{9}$

5. (10 points) Compute the volume integral

$$\int\int\int_E \overset{36}{\cancel{48}}\, x\, y\, z\, dV$$

where E is the region in 3D

$$\{(x, y, z)\,|\,0 \leq x \leq y \leq z \leq 1\} \quad .$$

The **type** of the answer is:

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ans.

$$\{(x,y,z) \,|\, 0 \leq x \leq y,\ 0 \leq y \leq z,\ 0 \leq z \leq 1\}$$

$$\int_0^1 \int_0^z \int_0^y 36xyz\, dx\, dy\, dz$$

$$\int_0^y 36xyz\, dx = 18y^3 z$$

$$\int_0^z 18y^3 z\, dy = \frac{9z^5}{2}$$

$$\int_0^1 \frac{9z^5}{2} dz = \frac{3}{4}$$

6. (10 points) By converting to polar coordinates, compute

$$\cancel{\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{(x^2+y^2)^2}{243\pi}\,dy\,dx} \qquad \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \frac{(x^2+y^2)^2}{243\pi}\,dy\,dx$$

The **type** of the answer is:

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ans.

$$x^2+y^2=4$$

$$x=2\cos\theta,\ y=2\sin\theta$$

$$\{(r,\theta) \mid 0\le r\le 2,\ 0\le\theta\le 2\pi\}$$

$$\int_0^{2\pi}\int_0^{2} \frac{r^4}{243\pi}\, r\,dr\,d\theta = \frac{64}{729}$$

7. (10 points) Compute the line integral

$$\cancel{\int_C \frac{4\sqrt{3}\,xyz}{3}\,ds} \quad , \qquad \int_C \frac{4\sqrt{5}\,xyz}{5}\,ds$$

where C is the line-segment joining $(0,0,0)$ and $(1,1,1)$

The **type** of the answer is:

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ans.

$r(t) = (1-t)\langle 0,0,0\rangle + t\langle 1,1,1\rangle$

$r(t) = \langle t,t,t\rangle, \quad 0 \le t \le 1$

$F(r(t)) = \dfrac{4\sqrt{5}\,t^3}{5}$

$r'(t) = \langle 1,1,1\rangle, \quad |r'(t)| = \sqrt{3}$

$$\int_0^1 \left(\frac{4\sqrt{5}}{5}t^3\right)\left(\sqrt{3}\right)dt = \frac{\sqrt{3}}{\sqrt{5}}$$

8. (10 points) Compute

$$\int_0^3 \int_{\sqrt{y/3}}^1 e^{x^3}\, dx\, dy \quad . \qquad \int_{0=y}^{2=y} \int_{\sqrt{y/2}=x}^{1=x} e^{x^2}\, dx\, dy$$

(Hint: Not even Dr. Z. can do $\int e^{x^3}\, dx$, so you must be clever, and first change the order of integration.)

The **type** of the answer is:

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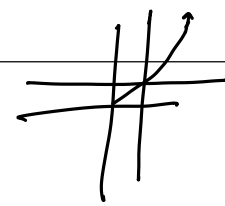
ans.



$$\int_{x_0}^{x} \int_{y_0}^{y} f(x,y)\, dy\, dx$$

$$\int_0^1 \int_0^{2x^2} e^{x^2}\, dy\, dx$$

$$\int_0^{2x^2} e^{x^2}\, dy = \left[y e^{x^2} \right]_0^{2x^2} = 2x^2 e^{x^2}$$

$$\int_0^2 2x^2 e^{x^2}\, dx = 92.74$$

9. (10 points) Compute the volume integral

$$\iiint_E \frac{\overset{7}{\cancel{5}}\,(x^2+y^2+z^2)}{4\pi}\,dV \quad ,$$

where

$$E = \{(x,y,z) \mid x^2 \; + \; y^2 \; + \; z^2 \leq 1\} \quad .$$

The **type** of the answer(s) is:

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ans.

$$\{(\rho, \phi, \theta) \mid 0 \leq \rho \leq 1,\ 0 \leq \phi \leq \pi,\ 0 \leq \theta \leq 2\pi\}$$

$$x^2 + y^2 + z^2 = \rho^2 \qquad dV = \rho^2$$

$$\frac{7}{4\pi}\int_0^1\int_0^\pi\int_0^{2\pi} \rho^2 \rho^2 \sin\phi \, d\theta \, d\phi \, d\rho = \frac{7}{4\pi}\int_0^1\int_0^\pi\int_0^{2\pi} \rho^4 \sin\phi \, d\theta \, d\phi \, d\rho$$

$$\frac{7}{4\pi}\left(\int_0^1 \rho^4 \, d\rho\right)\left(\int_0^\pi \sin\phi \, d\phi\right)\left(\int_0^{2\pi} d\theta\right)$$

$$\left(\frac{7}{4\pi}\right)\left(\frac{1}{7}\right)(2\pi)$$

$$\boxed{\frac{\pi}{2}}$$

10. (10 points) Find $\nabla \cdot \mathbf{F}$ if

$$\cancel{\mathbf{F} = \langle \sin(xy), \sin(yz), \sin(xz) \rangle}\ .$$

$$F = \langle \cos(xy), \cos(yz), \cos(xz) \rangle$$

The **type** of the answer is:

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ans.

$$F = \langle \underbrace{\cos(xy)}_{P}, \underbrace{\cos(yz)}_{Q}, \underbrace{\cos(xz)}_{R} \rangle$$

$$\nabla \cdot F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\boxed{\nabla \cdot F = -y\sin(xy) - z\sin(xy) - x\sin(xz)}$$