1. $x = uv\ y = uw\ z = vw$ *at the point* $(u,v,w) = (1,1,1)$

$$transformation = \begin{matrix} v & w & 0 \\ u & 0 & w \\ 0 & u & v \end{matrix} = \begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{matrix} = -1 - 1 = -2$$

2. $i)$ *show that* $F =< yz, xz, xy >$ *is a conservative vector field*

$$the\ curl\ is \begin{matrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yz & xz & xy \end{matrix} = (x - x)i - (y - y)j + (z - z)k = 0$$

$ii)\ f(x,y,z) = xyz$

$iii)$ *Find the line − integral* $\int_C F \cdot dr\ , r =< e^t, \cos(t-1)\ , t^5 >, 0 \le t \le 1$

the line − integral equals to $f(e,1,1) - f(1, -\cos(1)\ , 0) = e$

3. *change the order of integration* $\int_{-1}^{1}\int_{0}^{e^x} F(x,y)dy\, dx$

$$\int_0^{e^{-1}}\int_{-1}^{1} F(x,y)dx\, dy + \int_{e^{-1}}^{e}\int_{\ln(y)}^{1} F(x,y)dx\, dy$$

4. *Use Largange multiplier to find the smallest value that* $x^2 + y^2 + z^2$ *given that* $x + 3y + 5z = 35$

$$\begin{cases} 2x = \alpha \\ 2y = 3\alpha \\ 2z = 5\alpha \\ x + 3y + 5z = 35 \end{cases} \to \begin{cases} \alpha = 2 \\ x = 1 \\ y = 3 \\ z = 5 \end{cases} \to x^2 + y^2 + z^2 = 35$$

5. *compute the integral* $\iiint_E x^y * zdV\ \{(x,y,z)|0 \le y \le x \le z \le 1$

$$\int_0^1\int_0^z\int_0^x xyzdy\, dx\, dz = \frac{1}{48}$$

6. $\int_{-5}^{5}\int_{0}^{\sqrt{10-x^2}} 1dy\, dx = \int_0^{\pi}\int_0^5 rdr\, d\theta = \frac{25\pi}{2}$

7. $\int_C xyzds$ *where C is the line segment joining* $(1,2,3)$*and* $(4,7,6)$

$r(t) = (1 + 3t, 2 + 5t, 3 + 3t)\ \ r'(t) = (3,5,3)\ |r'(t)| = \sqrt{34}$

$\int_0^1 \sqrt{34}(1 + 3t)(2 + 5t)(3 + 3t)dt = \frac{251\sqrt{34}}{4}$

8. $compute \int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} dxdy = e - 1$

9. $\iiint_E 1dV, E = \{(x, y, z) | x^2 + y^2 + z^2 \leq 100\}$

$\int_0^{\pi} \int_0^{2\pi} \int_0^{10} \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi = \frac{4000\pi}{3}$

10. $Find\ \nabla \cdot F, if\ F = < xy, yz, xz >$

$$\nabla \cdot F = \frac{\partial(xy)}{\partial x} + \frac{\partial(yz)}{\partial y} + \frac{\partial(xz)}{\partial z} = x + y + z$$