11/22/22 2017 Exam 2 Q21

1) $x = 3u \cdot 3v + 3w = 9uv + 3w$

$y = 9uw + 3v$

$z = 9vw + 3u$ at point $(2,2,2) = (u,v,w)$

Find the Jacobian.

$$\det \begin{array}{c} \begin{pmatrix} 9v & 9u & 3 \\ 9w & 3 & 9u \\ 3 & 9w & 9u \end{pmatrix} \\ u \quad v \quad w \end{array} \quad \begin{pmatrix} 18 & 18 & 3 \\ 18 & 3 & 18 \\ 3 & 18 & 18 \end{pmatrix} = -8775$$

$=$

2) $F = \langle y^2 z^3, 2xyz^3, 3xy^2 z^2 \rangle$

Conservative? Calculate Curl:

$$\begin{pmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y^2 z^3 & 2xyz^3 & 3xy^2z^2 \end{pmatrix} = 0.$$

$\int \frac{df}{dx} = \int y^2 z^3$

$f = xy^2z^3 + g(y,z)$

$\int 2xyz^3 \, dy = xy^2z^3 + h(z)$

$\int 3xy^2z^2 \, dz = xy^2z^3$

$= xy^2z^3 = f(x,y,z)$

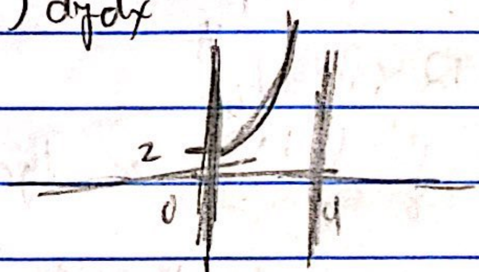
iii) $r = \langle \sin t, \cos t + 1, \sin 2t \rangle$ $\quad 0 \le t \le \pi$

$r(0) \rightarrow (0, 2, 0)$

$r(\pi) \rightarrow (0, 0, 0)$

$f(0,0,0) - f(0,2,0) = 0$

3) $\int_1^4 \int_0^{e^x+2} F(x,y)\, dy\, dx$



$1 \le x \le 4$

$0 \le y \le e^x + 2$

$\int_0^{e+1} \int_1^2 F(x,y) + \int_{e+1}^{e^4+2} \int_{\ln(y-1)}^{2} F(x,y)\, dx\, dy$

4) $xyz = 10$

Smallest $(x+y+z)$ can be

$f = x+y+z \qquad g = xyz$

$\nabla f = \langle 1,1,1 \rangle = \lambda \langle yz, xz, xy \rangle$

$1 = yz$

$1 = xy \qquad 1 = \frac{y}{z} \qquad z = y$

$1 = xz \qquad 1 = y^2;\ y = 1$

$x + y + z = 1 + 1 + 1 = 3$

5) $\iiint 24xyz\,dV \quad 0 \le x \le y \le z \le 1$

$0 \le z \le 1$

$\int_0^1 \int_0^z \int_0^y 24xyz\,dx\,dy\,dz \quad 0 \le y \le z$

$0 \le x \le y$

$\int_0^y 24xyz\,dx$

$12x^2yz\Big|_0^y$

$\int_0^1 \int_0^z 12y^3z\,dy\,dx$

$3y^4z\Big|_0^z$

$\int_0^1 3z^5\,dz$

$\frac{1}{2}z^6\Big|_0 = \frac{1}{2}$

6) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(\frac{x^2+y^2}{243\pi}\right)^2 dy\,dx$

$x^2+r^2=r^2$

$0 \le r \le 3 \quad \int_0^{2\pi} \int_0^1 \frac{r^4}{243\pi}\,r\,dr\,d\theta$

$\frac{1}{243\pi} \int_0^{2\pi} \frac{r^5}{5}\Big|_0^1 d\theta$

$\frac{1}{243\pi} \int_0^{2\pi} \frac{1}{5}\,d\theta$

$\frac{\cancel{6}}{5}\,\frac{1}{243\pi} \cdot 2\pi = \frac{2}{243}$

7) $\int_c \frac{8\sqrt{3}}{2} xyz\,ds$

from $(0,0,0)$ to $(3,3,3)$ $\longrightarrow$ $\langle 0,0,0\rangle + t\langle 3,3,3\rangle$

$= \langle 3t, 3t, 3t\rangle$

$4\sqrt{3}\, xyz$

$\cancel{t\le 0} \le 0 \le 1 \le t$

$r'(t) = \langle 3,3,3\rangle$

$\sqrt{27} = 3\sqrt{3}$

$\int_0^1 4\sqrt{3}\cdot\sqrt{3}\cdot(3t)^3\,dt$

$\int_0^1 (27t^3 \cdot 3 \cdot 4)\,dt$

$\frac{324t^4}{4}\Big|_0^1 = 81$

8) $\int_0^3 \int_{\sqrt{y/3}}^1 3e^{x^3}\,dx\,dy$

$0 \le y \le 3 \quad \sqrt{y/3} \le x \le 1$

$3\int_0^1 \int_0^{3x^2} e^{x^3}\,dy\,dx$

$e^{x^3}(y)\Big|_0^{3x^2} = 3\int_0^1 3x^2 e^{x^3}\,dx$

$= 3(e-1)$

9) $\iiint \frac{1}{4\pi}(x^2+y^2+z^2)\,dV$

$x^2+y^2+z^2 \le 9$

$r=3$

$0 \le \rho \le 3$

$0 \le \phi \le \pi$

$0 \le \theta \le 2\pi$

$\frac{1}{4\pi}\int_0^3 \int_0^{\pi} \int_0^{2\pi} \rho^4 \sin\phi \, d\theta \, d\phi \, d\rho$

$= \frac{1}{4\pi}\int_0^3 \rho^4 \, d\rho \cdot \int_0^{\pi} \sin\phi \, d\phi \cdot \int_0^{2\pi} d\theta$

$\frac{1}{4\pi}\left(\frac{r^5}{5}\Big|_0^1\right) \cdot \left(-\cos\phi \Big|_0^{\pi}\right) \cdot \int_0^{2\pi} d\theta$

$\frac{1}{20\pi}(1+1)(2\pi)1 = \frac{1}{5}$

10) $F = 3\langle \cos(xy), \cos(yz), \cos(xz) \rangle$

→ Divergence

$\nabla \cdot F = \frac{d}{dx}(3\cos xy) + \frac{d}{dy}(3\cos(yz)) + \frac{d}{dz}(3\cos yz)$

$= -3(x\sin(xz) + y\sin(xy) + z\sin(yz))$.