1. Find the Jacobian of the transformation from (u, v, w)-space to (x, y, z)-space. $x = u^2$, $y = v^2$, $z = w^2$. at the point $(u, v, w) = (1, 1, 31)$.

$$\det\begin{pmatrix} 2u & 0 & 0 \\ 0 & 2v & 0 \\ 0 & 0 & 2w \end{pmatrix}$$

$$= \det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= 1$$

2. $F = \langle e^x + yz, e^y + xz, e^z + xy \rangle$ $r = \langle e^t, 1+et, e^{2t} \rangle$, $0 \le t \le 1$

(i) $\frac{d}{dx} = e^x$, $\frac{d}{dy} = e^y$, $\frac{d}{dz} = e^z$

$$\det\begin{pmatrix} i & j & k \\ e^x & e^y & e^z \\ e^x+yz & e^y+xz & e^z+xy \end{pmatrix}$$

$= i(e^y \cdot (e^z+xy) - e^z \cdot (e^y+xz)) - j(e^x \cdot (e^z+xy) - e^z \cdot (e^x+yz)) + k(e^x \cdot (e^y+xz) - e^y \cdot (e^x+yz))$

$= \langle 0, 0, 0 \rangle$

(ii). $f_x = e^x + yz$, $f_y = e^y + xz$, $f_z = e^z + xy$

$f = \int (e^x + yz)\,dx = e^x + xyz + \phi(y, z)$

$\cancel{e^x + xyz + \phi(y,z) = e^y + xz}$

$e^y + xz + \phi(yz) = e^y + xz$

$f_z = e^z + xy$

$e^z + xy + \psi'(z) = e^z + xy$

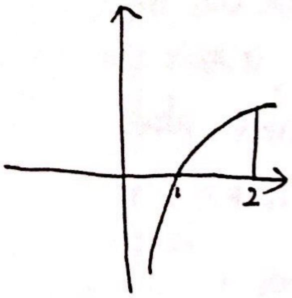
$f(x, y, z) = e^x + e^y + e^z + xyz$

(iii) $\int_C F \cdot dr = \cancel{f(1,2,1) - f}\ f(e, 1+e, e^2) - f(1, 2, 1)$

$=$

3. $\int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx$

$R = \{(x,y) \mid 0 \le y \le \ln x,\ 1 \le x \le 2\}$



$R = \{(x,y) \mid e^y \le x \le 2,\ 0 \le y \le \ln 2\}$

$\int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\,dx\,dy$

4. $f(x,y,z) = xyz$ where $x>0, y>0, z>0$

$x+y+z-51=0$, find the maximum

$\nabla f = yz\,i + xz\,j + xy\,k$

$g(x,y,z) = x+y+z = 51$

$\nabla g = i+j+k$

$yz = \lambda,\ xz = \lambda,\ xy = \lambda$

$\therefore\ x = y = z$

$3x = 51$

$x = 17$

$f(x,y,z) = 4913$

5. $\int_0^1 \int_0^1 \int_0^1 xyz\,dV$

$= \int_0^1 \int_0^1 \int_0^1 xy+z\,dx\,dy\,dz$

$= \int_0^1 \int_0^1 (z + y\frac{1}{2})\,dy\,dz$

$= \int_0^1 (z + \frac{1}{4})\,dz$

$= \frac{3}{4}$

6. $\int_0^1 \int_x^{\sqrt{2-x^2}} 1 \, dy \, dx$

$y=x$, $y=\sqrt{2-x^2} \Rightarrow y^2+x^2=2$

$x=0$ and $x=1$

$x=r\cos\theta$, $y=r\sin\theta$

$dxdy = r dr d\theta$

r: 0 to $\sqrt{2}$, θ: $\frac{\pi}{4}$ to $\frac{\pi}{2}$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\sqrt{2}} 1 \, dr \, d\theta$

$= \frac{\pi}{2\sqrt{2}}$

7. $\int_C (x^2-y+3z) ds$ from $(0,0,0)$ to $(1,2,1)$

$\langle x, y, z\rangle = t\langle 1, 2, 1\rangle = \langle t, 2t, t\rangle$

$ds = \sqrt{1^2+2^2+1^2} dt = \sqrt{6} \, dt$

$\sqrt{6}\int_0^1 (t^2+t) dt$

$= \frac{5}{\sqrt{6}}$

8. $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1} dy \, dx$

$= \int_0^1 \int_0^{x^{3/2}} ye^{x^2} dy dx$

$= \frac{1}{2}\int_0^1 e^{x^2} x^3 dx$

$= \frac{1}{4}$

9. $\iiint_E (x^2+y^2+z^2)^2 \, dV$

$E = \{(x,y,z) \mid x^2+y^2+z^2 \leq 2\}$

$\{(\rho, \phi, \theta) \mid 0 \leq \rho \leq 1, \ 0 \leq \phi \leq \pi, \ 0 \leq \theta \leq 2\pi\}$

$= \int_0^1 \rho^6 \, d\rho \cdot \int_0^{\pi} \sin\phi \, d\phi \cdot \int_0^{2\pi} d\theta$

$= \frac{1}{6}(-(-1)-(-1))2\pi = \frac{2\pi}{3}$

10. $\vec{F} = \langle x^2yz, \ xy^2z, \ xyz^2 \rangle$

$\nabla \cdot \vec{F} = 2xyz + 2xyz + 2xyz$