Mt2 Make-up.

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Section: 23.

1. Find the Jacobian of the transforming from (u, v, w)-space to (x, y, z)-space.

$$x = uv + wv \qquad y = uw + vu \qquad z = vw + uw$$

at the point $(u, v, w) = (\cancel{2,2,2})$ 3, 1, 5.

Type: Number.

$$\begin{vmatrix} v & u+w & v \\ v+w & u & u \\ w & w & v+u \end{vmatrix} = v(uv+u^2-uw) - (u+w)(v^2+wv+uv+wu) + v(vw+w^2-uw)$$

$$= (3+9-(15)) - 8\times(1+5+3+15) + 5+25-15$$

$$= -180.$$

2. (i) Show that $F = \langle e^x + yz,\ e^y + xz,\ e^z + xy \rangle$ is conservative

(ii) Find a function $f(x, y, z)$ such that $F = \nabla f$

(iii) Find the line-integral $\int_C F \cdot dr$ where C is the curve

$\cancel{r = \sin t, \cos t,}$ $r = \langle t^2, 1-t, 2t \rangle$, $0 \le t \le 2$.

(i) $\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^x+yz & e^y+xz & e^z+xy \end{vmatrix} = i(x-x) - j(y-y) + k(z-z) = 0.$

$\therefore$ It is conservative

(ii) For x, $f = e^x + xyz + g(y, z)$

For y. $f = e^x + e^y + xyz + h(z)$.

For z. $f = e^x + e^y + e^z + xyz$.

Type: Multivariable function

(iii) $r_{start} = \langle 0, 1, 0 \rangle$ $r_{end} = \langle 4, -1, 4 \rangle$.

$f(4, -1, 4) - f(0, 1, 0) = 2e^4 + e^{-1} - 18 - e$

Type: Number.

3. Sketch the region of integration and change the order of integration

$$\int_0^2 \int_0^{x^3} F(x, y)\, dy\, dx.$$

$0 \le y \le x^3$ $\quad$ $0 \le x \le 2$ $\quad$ $\cancel{1 \le x \le 8}$

Type: abstract double integral.

$0 \le y \le 8$ $\quad$ $y^{\frac{1}{3}} \le x \le 2$.

$$\int_0^8 \int_{y^{\frac{1}{3}}}^2 F(x, y)\, dx\, dy$$

4. Use Largange multipliers to find the ~~smallest~~ largest value that $(x+3y+5z)$ can be, given that $x^2+y^2+z^2=35$

Type: Number.

$$\nabla f = \langle 1, 3, 5 \rangle$$

$$\nabla g = \langle 2x, 2y, 2z \rangle$$

$$\begin{cases} 1 = \lambda 2x \\ 3 = \lambda 2y \\ 5 = \lambda 2z \end{cases}$$

$$\frac{1}{4\lambda^2} + \frac{9}{4\lambda^2} + \frac{25}{4\lambda^2} = 35.$$

$$\lambda^2 = \frac{1}{4}. \qquad \begin{cases} x = \frac{1}{2\lambda} \\ y = \frac{3}{2\lambda} \\ z = \frac{5}{2\lambda} \end{cases} \qquad x+3y+5z = \frac{35}{2\lambda}.$$

$$\lambda = \pm\frac{1}{2}$$

$\lambda = \frac{1}{2}$, $x+3y+5z = 35$,

$\lambda = -\frac{1}{2}$, $x+3y+5z = -35$

$\therefore$ The largest value is 35.

5. Compute the volume integral

$$\iiint_E \cancel{6x^2y} \; 6x^2y^2z^2 \, dV$$

where E is the region in 3D $\{(x,y,z) \mid 0 \le x \le y \le z \le 36\}$.

~~The~~ Type: Number.

$$\int_0^{36} \int_0^{z} \int_0^{y} 6x^2y^2z^2 \, dx\,dy\,dz$$

Inner Loop: $\left[2x^3y^2z^2\right]_0^y = 2y^5z^2$

Middle Loop: $\left[\frac{1}{3}y^6z^2\right]_0^z = \frac{1}{3}z^8$

Outer Loop: $\left[\frac{1}{27}z^9\right]_0^{36} = 3761479876608$

6. By converting to polar coordinates compute

$$\int_{-6}^{6}\int_{-\sqrt{36-x^2}}^{\sqrt{36-x^2}} \frac{x^2+y^2}{15}\,dy\,dx$$

Type: Number.

$0 \le r \le 6 \quad 0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^6 \frac{r^3}{15}\,dr\,d\theta$$

Inner Loop: $\left[\frac{r^4}{60}\right]_0^6 = \frac{108}{5}$

Outer Loop: $2\pi \times \frac{108}{5} = \frac{216}{5}\pi$.

7. Compute the line integral

$\int_C 10x^2y^2z^2\,ds$ C is the ~~seg~~ line-segment from $(0,0,0)$ to $(3,3,3)$

Type: Number

$r(t) = \langle 3t, 3t, 3t \rangle \quad 0 \le t \le 1.$

$|r'(t)| = \sqrt{3^2+3^2+3^2} = 3\sqrt{3}$

$$\int_0^1 10 \times 9t^2 \times 9t^2 \times 9t^2 \times 3\sqrt{3}\,dt = \int_0^1 21870\sqrt{3}\,t^6\,dt = \frac{21870\sqrt{3}}{7}$$

8. Compute ~~the~~ $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\,dx\,dy$

Type: Number

$0 \le y \le 2 \quad \frac{y}{2} \le x \le 1.$

$0 \le y \le 2x \quad 0 \le x \le 1.$

$$\int_0^1 \int_0^{2x} e^{x^2}\,dy\,dx.$$

Inner Loop: $\left[ye^{x^2}\right]_0^{2x} = 2xe^{x^2}$

Outer loop: $\left[e^{x^2}\right]_0^1 = e-1.$

9. Compute the volume integral

$$\iiint_E x^2 \, dV$$

where $E = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 9\}$

Type: Number.

$0 \leq \rho \leq 3, \quad 0 \leq \phi \leq \pi, \quad 0 \leq \theta \leq 2\pi$

$$\int_0^{\pi} \int_0^{2\pi} \int_0^3 \rho^4 \sin^3\phi \cos^2\theta \, d\rho \, d\theta \, d\phi$$

Inner Loop: $\left[\frac{1}{5}\rho^5 \sin^3\phi \cos^2\theta\right]_0^3 = \frac{243}{5} \sin^3\phi \cos^2\theta.$

Middle Loop: $\left[\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\frac{243}{5}\sin^3\phi\right]_0^{2\pi} = \frac{243}{5}\pi \sin^3\phi$

Outer Loop: $\left[\frac{243\pi}{5}\left(-\cos\phi + \frac{\cos^3\phi}{3}\right)\right]_0^{\pi} = \frac{4}{3} \times \frac{243}{5}\pi = \frac{324}{5}\pi$

10. Find ∇F, if

$$F = \langle \sin x^2, \sin y^2, \sin z^2 \rangle$$

Type: Multivariable function.

$$\nabla F = 2x \cos x^2 + 2y \cos y^2, 2z \cos z^2.$$