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MATH 251 (4,6,7), Dr. Z. , Exam 2, Tue., Nov. 21, 2017, SEC 118

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM

Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

MAKE SURE TO PUT THE TYPE!

Types: Number, Function of *variable*(s), 2D vector of numbers, 3D vector of numbers, 2D vector of functions (aka 2D vector-field), 3D vector of functions (aka 3D vector field), equation of a plane, parametric equation of a line, equation of a line, equation of a surface, equation of a line, DNE (does not exist), abstract double-integral, abstract triple-integral.

1. (10 pts.)
Find the Jacobian of the transformation from (u, v, w)-space to (x, y, z)-space.

$$x = uv + w \quad , \quad y = uw + v \quad , \quad z = vw + u \quad ,$$

at the point $(u, v, w) = \cancel{(2, 2, 2)}$. $(3,3,3)$.

The **type** of the answers is: number

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ans.

-28

$$\det\begin{pmatrix} v & u & 1 \\ w & 1 & u \\ 1 & w & v \end{pmatrix}$$

$$(u,v,w) = (3,3,3)$$

$$\det\begin{pmatrix} 3 & 3 & 1 \\ 3 & 1 & 3 \\ 1 & 3 & 3 \end{pmatrix} = \boxed{-28}$$

2. (10 points altogether) *(not changing numbers to keep conservative vector field)

(i) (3 points) Show that

$$\mathbf{F} = \langle\, 3\,x^2yz{+}yz{+}\cos\,(x+y+z)\ ,\ x^3z{+}xz{+}\cos\,(x+y+z)\ ,\ x^3y{+}xy{+}\cos\,(x+y+z)\,\rangle \quad ,$$

is a conservative vector field.

(ii) (4 point) Find a function $f(x,y,z)$ such that $\mathbf{F} = \bigtriangledown f$.

(iii) (3 points) Find the line-integral $\int_C \mathbf{F}.d\mathbf{r}$ where C is the curve

$$\mathbf{r} = \langle \sin t, \cos t + 1, \sin 2t \rangle \quad , \quad 0 \le t \le \pi \quad .$$

The **types** of the answer is: For (ii) equation For (iii) number

answers (ii) f(x,y,z)= $x^3yz + xyz + \sin(x+y+z)$

(iii) $-\sin(2)$

(i) $\nabla \times F = \det\begin{pmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 3x^2yz+yz+\cos(x+y+z) & x^3z+xz+\cos(x+y+z) & x^3y+xy+\cos(x+y+z) \end{pmatrix} = \langle 0,0,0 \rangle$

Thus, it's conservative.

(ii) $f = \int (3x^2yz + yz + \cos(x+y+z))\,dx = x^3yz + xyz + \sin(x+y+z) + \phi(y,z)$

$f(x,y,z) = x^3yz + xyz + \sin(x+y+z)$

(iii) $f(0,0,0) - f(0,2,0) = (0 + \sin(0)) - (0 + \sin(2)) = -\sin(2)$

3. (10 points)
Sketch the region of integration and change the order of integration.

$$\int_1^2 \int_0^{e^x+1} F(x,y)\,dy\,dx$$

The **type** of the answer is: sum of two double integrals

ans.

$$\int_0^{e+1}\int_1^2 F(x,y)\,dx\,dy + \int_{e+1}^{e^2+1}\int_{\ln(y-1)}^{2} F(x,y)\,dx\,dy$$

$\{(x,y) \mid 1 \le x \le 2,\ 0 \le y \le e^x+1\}$

$\{(x,y) \mid 1 \le y \le e+1,\ 1 \le x \le 2\} \cup \{(x,y) \mid e+1 \le y \le e^2+1,\ \ln(y-1) \le x \le 2\}$

$$\int_0^{e+1}\int_1^2 F(x,y)\,dx\,dy + \int_{e+1}^{e^2+1}\int_{\ln(y-1)}^{2} F(x,y)\,dx\,dy$$

4. (10 points) Use Largange multipliers (no credit for other methods) to find the smallest value that $x+y+z$ can be, given that ~~$xyz = 1$~~. $xyz = 125$.

The **type** of the answer is: number

ans.

-15

$\to \nabla f = \langle 1,1,1 \rangle$ and $\nabla g = \langle yz, xz, xy \rangle$

$\to \langle 1,1,1 \rangle = \lambda \langle yz, xz, xy \rangle \Rightarrow \langle 1,1,1 \rangle = \langle \lambda yz, \lambda xz, \lambda xy \rangle$

$\to 1 = \lambda yz, \quad 1 = \lambda xz, \quad 1 = \lambda xy$

$\to 1 = \lambda^3 x^2 y^2 z^2$

$\to \frac{1}{\lambda^3} = (xyz)^2$

$\to \frac{1}{\lambda^3 x^2} = y^2 z^2 \Rightarrow \frac{1}{\lambda^2} = \frac{1}{\lambda^3 x^2} \Rightarrow \lambda^3 x^2 = \lambda^2 \Rightarrow x^2 = \frac{1}{\lambda} \Rightarrow x = \pm \frac{1}{\sqrt{\lambda}}$

$\to y = \pm \frac{1}{\sqrt{\lambda}}, \quad z = \pm \frac{1}{\sqrt{\lambda}}$

$\to \frac{1}{\sqrt{\lambda}} \cdot \frac{1}{\sqrt{\lambda}} \cdot \frac{1}{\sqrt{\lambda}} = 125 \Rightarrow \frac{1}{\lambda^{3/2}} = 125 \Rightarrow \lambda = \frac{1}{25}$

$\to x = \pm 5, \; y = \pm 5, \; z = \pm 5$

$\to -5-5-5 = -15$

$\to \boxed{-15 \text{ is the smallest value}}$

5. (10 points) Compute the volume integral

$$\int\int\int_E 48\, x\, y\, z\, dV$$

where E is the region in 3D

$$\{(x, y, z)\,|\,0 \leq x \leq y \leq z \leq 1\} \quad .$$

The **type** of the answer is: number

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ans. 1

$\{(x,y,z) \mid 0 \leq z \leq 1,\ 0 \leq y \leq z,\ 0 \leq x \leq y\}$

$\int_0^1 \int_0^z \int_0^y (48xyz)\, dx\, dy\, dz = \boxed{1}$

6. (10 points) By converting to polar coordinates, compute

$$\int_{-3}^{3}\int_{-\cancel{\sqrt{9-x^2}}\ -\sqrt{4-x^2}}^{\cancel{\sqrt{9-x^2}}\ \sqrt{4-x^2}} \frac{\left(x^2+y^2\right)^2}{243\pi}\,dy\,dx$$

The **type** of the answer is: number

ans. $\frac{64}{729}$

$\{(r,\theta) \mid 0 \le r \le 2,\ 0 \le \theta \le 2\pi\}$

$$\int_0^{2\pi}\int_0^{2} \frac{r^4}{243\pi}\, r\,dr\,d\theta = \frac{2}{243}\left(\frac{r^6}{6}\Big|_0^2\right) = \frac{64}{6}\cdot\frac{2}{243} = \boxed{\frac{64}{729}}$$

7. (10 points) Compute the line integral

$$\int_C \frac{4\sqrt{3}\,x\,y\,z}{\cancel{3}}\,ds \quad ,$$

where C is the line-segment joining $(0,0,0)$ and $(1,1,1)$

The **type** of the answer is: number

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ans.

3

$\langle 0,0,0\rangle + t\left(\langle 1,1,1\rangle - \langle 0,0,0\rangle\right) = \langle t,t,t\rangle;\ 0 \le t \le 1$

$r'(t) = \langle 1,1,1\rangle;\ \|r'(t)\| = \sqrt{3}$

$\int_0^1 4\sqrt{3}\,t^3\sqrt{3}\,dt = 12\int_0^1 t^3\,dt = 12\left[\frac{t^4}{4}\Big|_0^1\right] = 12\cdot\frac{1}{4} = \boxed{3}$

8. (10 points) Compute

$$\int_0^3 \int_{\sqrt{y/3}}^1 e^{x^3}\, dx\, dy \quad .$$

(Hint: Not even Dr. Z. can do $\int e^{x^3}\, dx$, so you must be clever, and first change the order of integration.)

The **type** of the answer is: number

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ans.

$e-1$

$\{(x,y) \mid 0 \le y \le 3,\ \sqrt{\frac{y}{3}} \le x \le 1\} \Rightarrow \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 3x^2\}$

$\int_0^1 \int_0^{3x^2} e^{x^3}\, dy\, dx = \boxed{e-1}$

9. (10 points) Compute the volume integral

$$\int\int\int_E \frac{5\,(x^2+y^2+z^2)}{4\pi}\,dV \quad ,$$

where

$$E = \{(x,y,z)\,|\,x^2\,+\,y^2\,+\,z^2 \le 1\} \quad .$$

The **type** of the answer(s) is: number

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ans. 1

$\{(\rho, \phi, \theta)\,|\,0 \le \rho \le 1,\ 0 \le \phi \le \pi,\ 0 \le \theta \le 2\pi\}$

$\frac{5}{4\pi}\int_0^1\int_0^{\pi}\int_0^{2\pi} \rho^4 \sin\phi \, d\theta \, d\phi \, d\rho = \boxed{1}$

10. (10 points) Find $\nabla \cdot \mathbf{F}$ if

$$\mathbf{F} = \langle \cancel{\sin(xy)}\,, \cancel{\sin(yz)}\,, \cancel{\sin(xz)} \rangle \quad .$$

$-\cos(xy)$ $-\cos(yz)$ $-\cos(xz)$

The **type** of the answer is: equation

ans.

$y\sin(xy) + z\sin(yz) + x\sin(xz)$

$$\nabla \cdot F = \frac{\partial}{\partial x}\left(-\cos(xy)\right) + \frac{\partial}{\partial y}\left(-\cos(yz)\right) + \frac{\partial}{\partial z}\left(-\cos(xz)\right)$$

$$\nabla \cdot F = \boxed{y\sin(xy) + z\sin(yz) + x\sin(xz)}$$