Attendance Quiz
Exam Review

1. Find Jacobian:

$x = 2uv + 2w \qquad y = 2uw + 2v \qquad z = 2vw + 2u$

at $(u,v,w) = (2,2,2)$

$\frac{\partial x}{\partial u} = 2v \qquad \frac{\partial x}{\partial v} = 2u \qquad \frac{\partial x}{\partial w} = 2$

$\frac{\partial y}{\partial u} = 2w \qquad \frac{\partial y}{\partial v} = 2 \qquad \frac{\partial x}{\partial w} = 2u$

$\frac{\partial z}{\partial u} = 2 \qquad \frac{\partial z}{\partial v} = 2w \qquad \frac{\partial z}{\partial w} = 2v$

$$\text{Det: } \begin{vmatrix} 4 & 4 & 2 \\ 4 & 2 & 4 \\ 2 & 4 & 4 \end{vmatrix} = 4(8-16) - 4(16-8) + 2(16-4)$$

$$-\cancel{32}\ \cancel{32} + 24 = \boxed{24}$$

2. Show that:

$F = \langle 6x^2yz + 2yz + 2\cos(x+y+z),\ 2x^3z + 2xz + 2\cos(x+y+z),\ 2x^3y + 2xy + 2\cos(x+y+z) \rangle$

is conservative

Find curl: $\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{matrix}$

$\hat{i}[\partial/\partial y(R) - \partial/\partial z(Q)] - \hat{j}[\partial/\partial x(R) - \partial/\partial z(P)] + \hat{k}[\partial/\partial x(Q) - \partial/\partial y(P)]$

$\hat{i}[2x^3 + 2x - 2\sin(x+y+z) - [2x^3 + 2x - 2\sin(x+y+z)] = 0$

$\hat{j}[6x^2y + 2y - 2\sin(x+y+z) - [6x^2y + 2y - 2\sin(x+y+z)]] = 0$

$k[6x^2z + 2z - 2\sin(x+y+z) - [6x^2z + 2z - 2\sin(x+y+z)] = 0$

This is the zero vector. $\langle 0,0,0 \rangle$

therefore it is conservative

Find a function $f(x,y,z)$ such that $F = \nabla f$

$$\frac{\partial f}{\partial x} = 6x^2yz + 2yz + 2\cos(x+y+z)$$

$$\frac{\partial f}{\partial y} = 2x^3z + 2xz + 2\cos(x+y+z)$$

$$\frac{\partial f}{\partial z} = 2x^3y + 2xy + 2\cos(x+y+z)$$

$$f = \int 6x^2yz + 2yz + 2\cos(x+y+z)\,dx = 2x^3yz + 2xyz + 2\sin(x+y+z) + g(y,z)$$

$$\frac{\partial f}{\partial y} = 2x^3z + 2xz + 2\cos(x+y+z) + \frac{\partial g}{\partial y} = 2x^3z + 2xz + 2\cos(x+y+z)$$

$$\frac{\partial g}{\partial y} = 0$$

$$\int 0\,dy = 0 + h(z)$$

$$\frac{\partial f}{\partial z} = 2x^3y + 2xy + 2\cos(x+y+z) + \frac{\partial h}{\partial z} = 2x^3y + 2xy + 2\cos(x+y+z)$$

$$\frac{\partial h}{\partial z} = 0$$

Final: $2x^3yz + 2xyz + 2\sin(x+y+z)$

(iii) Find line integral $\int_C F \cdot dr$ where C is the curve

$r = \langle \sin t, \cos t + 1, \sin 2t \rangle \qquad 0 \le t \le \pi$

$r(\pi) = \langle 0, 0, 0 \rangle \qquad r(0) = \langle 0, 2, 0 \rangle$

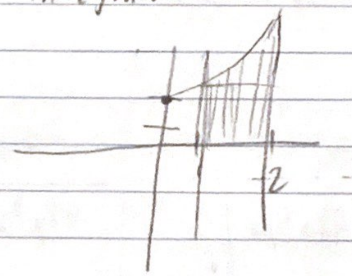
$[2(0 \cdot 0 \cdot 0) + 2(0) + 2\sin(0)] - [0 + 0 + 2\sin(2)]$

$-2\sin(2)$

3. Sketch the region of integration and change the order of integration

$$\int_1^2 \int_0^{e^x+1} F(x,y)\, dy\, dx$$

Sum of two integrals



$$\int_0^{e+1} \int_1^2 F(x,y)\, dx\, dy + \int_{e+1}^{e^2+1} \int_{\ln(y-1)}^{2} F(x,y)\, dx\, dy$$

First rectangle   Second part

4. Use Lagrange Multipliers

$2x+2y+2z$ $2xyz=2$

$\nabla f = \langle 2,2,2 \rangle$ $\nabla g = \langle 2yz, 2xz, 2xy \rangle$

$\lambda \nabla f = \nabla g$

$2\lambda = 2yz$ $2\lambda = 2xz$ $2\lambda = 2xy$ $2xyz = 2$

Using Maple to solve the system.

$L=1$ $x=1$ $y=1$ $z=1$

$2 \cdot 4 \cdot 2 \cdot 1 \cdot 2 = 36$

5. Compute the Volume Integral

$$\iiint_E 96xyz \, dV$$

where $E: \{(x,y,z) \mid 0 \le x \le y \le z \le 2\}$

$$\int_0^2 \int_0^z \int_0^y 96xyz \, dx\, dy\, dz$$

First inner: $\frac{96x^2yz}{2}\Big|_0^y = 48x^2yz\Big|_0^y = 48y^3z$

$$\int_0^2 \int_0^z 48y^3z \, dy\, dz = \frac{48y^4}{4}\Big|_0^z = 12z^5$$

$$\int_0^2 12z^5 \, dz = \frac{12z^6}{6} = 2z^6\Big|_0^2 = \boxed{128}$$

6. By converting to polar coordinates, compute

$$\int_{-6}^{6} \int_{-\sqrt{36-x^2}}^{\sqrt{36-x^2}} \frac{(x^2+y^2)^2}{243\pi} \, dy\, dx$$

$\{(R,\theta) \mid 0 \le R \le 6,\ 0 \le \theta \le 2\pi\}$

$$\frac{1}{243\pi} \int_0^{2\pi} \int_0^6 r^5 \, dr\, d\theta = \frac{r^6}{6}\Big|_0^6 =$$

$$\frac{1}{141\pi} \quad \frac{32}{\pi} \int_0^{2\pi} d\theta = \frac{32 \cdot 2\pi}{\pi}$$

$\boxed{64}$

7. Compute the line integral

$$\int_C \frac{8\sqrt{3}\,xyz}{3}\,dS$$

where C is the line segment joining $\underset{P}{(0,0,0)}$ and $\underset{Q}{(2,2,2)}$

$(1-t)\cdot P + t\cdot Q$

$(1-t)\cdot\langle 0,0,0\rangle + t\langle 2,2,2\rangle$

$\langle 0,0,0\rangle + \langle 2t,2t,2t\rangle = \langle \underset{x}{2t}, \underset{y}{2t}, \underset{z}{2t}\rangle \quad 0\le t\le 1$

$x'(t)=2 \qquad y'(t)=2 \qquad z'(t)=2$

$dS = \sqrt{4+4+4} = \sqrt{12}\,dt$

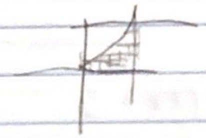
$$\int_0^1 \frac{8\sqrt{3}\,(2t)(2t)(2t)}{3}\cdot\sqrt{12}\,dt = \int_0^1 \frac{8\sqrt{3}\cdot 8t^3\cdot\sqrt{12}}{3}\,dt$$

$$\frac{8\sqrt{3}\cdot\sqrt{12}}{3}\cdot\int_0^1 8t^3\,dt = \frac{8\sqrt{3}\cdot\sqrt{12}}{3}\cdot\frac{8t^4}{4}\Big|_0^1 = \boxed{\frac{16\sqrt{3}\sqrt{12}}{3}} = \boxed{32}$$

8. Compute.

$$\int_0^3\int_{\sqrt{y/3}}^1 e^{x^3}\,dx\,dy$$

Change order of integration.



$$\int_0^1\int_0^{3x^2} e^{x^3}\,dy\,dx$$

$x = \sqrt{\frac{y}{3}}$

$x^2 = \frac{y}{3} \qquad y = 3x^2$

$e^{x^3}y\Big|_0^{3x^2} = 3e^{x^3}x^2$

$$\int_0^1 3e^{x^3}x^2\,dx$$

$u = x^3 \quad du = 3x^2\,dx \quad dx = \frac{du}{3x^2}$

$$\int \cancel{3x^2}e^u\frac{du}{\cancel{3x^2}} \qquad e^u = e^{x^3}\Big|_0^1 = \boxed{e-1}$$

9. Compute the volume integral

$$\iiint_E \frac{10(x^2+y^2+z^2)}{4\pi}\, dV$$

where $E = \{(x,y,z) \mid x^2+y^2+z^2 \le 1\}$

Spherical Coordinates

$\rho = R$ $\quad E = \{0 \le \rho \le 1, 0 \le \theta \le 2\pi, 0 \le \phi \le \pi\}$

$dV = \rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \frac{10(\rho^2)}{4\pi} \rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$$

$$\frac{10}{4\pi} \cdot \int_0^{\pi}\int_0^{2\pi}\int_0^{1} \rho^4 \sin(\phi)\, d\rho\, d\theta\, d\phi$$

$$\int_0^1 \rho^4 \sin(\phi)\, d\rho = \frac{\rho^5}{5} \sin(\phi)\Big|_0^1 = \frac{\sin(\phi)}{5}$$

$$\int_0^{2\pi} \frac{\sin(\phi)}{5}\, d\theta = \frac{\theta \sin(\phi)}{5}\Big|_0^{2\pi} = \frac{2\pi \sin(\phi)}{5}$$

$$\frac{2\cancel{0}}{\cancel{2}} \cdot \int_0^{\pi} \sin(\phi)\, d\phi = -\cos(\phi)\Big|_0^{\pi} = 1-(-1) = \boxed{2}$$

10. Find $\nabla \cdot F$ if

$F = \langle 2\sin(xy), 2\sin(yz), 2\sin(xz) \rangle$

This is the divergence of F

$P = 2\sin(xy)$ $\quad Q = 2\sin(yz)$ $\quad R = 2\sin(xz)$

$$\text{div}F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial y}$$

$y2\cos(xy) + z2\cos(yz) + x2\cos(xz)$