Q21 Aditya Sivakumar Calc 251 Section 24

1. Find Jacobian @ (1,1,1)

$x = u+v+w$
$y = u+w$
$z = v$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 1(-1) - 1(0) + 1(1) = \boxed{0}$$

2. $F = \langle 8x, 3, -1 \rangle$

i. $\text{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 8x & 3 & -1 \end{vmatrix}$ $i(0-0) - j(0-0) + k(0-0)$

$= 0$, conservative

ii. $f(x,y,z) = \int 8x\,dx = 4x^2 + g(y,z)$

$g_y(y,z) = 3$

$g(y,z) = 3y + h(z)$ $\therefore \boxed{f(x,y,z) = 4x^2 + 3y - z}$

$f(x,y,z) = 4x^2 + 3y + h(z)$

$h'(z) = -1$

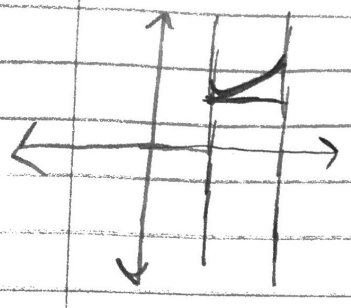
$z = -z$

iii. $\int_C F \cdot dr = f(r(1)) - f(r(0))$

$r(1) = \langle 0,0,0 \rangle$ $= 0 - 6 = \boxed{-6}$

$r(0) = \langle 0,2,0 \rangle$

3. Original: $\int_2^4 \int_0^{2e^x+2} f(x,y)\,dy\,dx$



$$\int_{2e^2+2}^{2e^4+2} \int_{\ln(\frac{y-2}{2})}^{4} f(x,y)\,dx\,dy$$

4. $4x+4y+4z \quad \because 2xyz=1$

$\nabla f = \nabla g\,\lambda$

$\langle 4,4,4\rangle = \lambda\langle 2yz, 2xz, 2xy\rangle$

$4=2yz \quad yz=xz=xy \quad 2x^3=1$

$4=2xz \quad x=y=z \quad x^2=\frac{1}{2}$

$4=2xy \quad \therefore \quad x=y=z=\frac{1}{\sqrt[3]{2}}$

$x+y+z = 3\left[\frac{4}{\sqrt[3]{2}}\right] = \boxed{\frac{12}{\sqrt[3]{2}}}$

5. $\iiint_E 10x^2yz\,dV$

$0\le x\le y\le z\le 1$

$0\le x\le 4$

$0\le y\le z$

$0\le z\le 1$

$10\int_0^1 \int_0^z \int_0^z x^2yz\,dx\,dy\,dz$

$= \frac{10}{3}\int_0^1 \int_0^z y^4z\,dy\,dz$

$= \frac{2}{3}\int_0^1 z^6\,dz = \boxed{\frac{2}{21}}$

6. $\int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \frac{10}{12}(x^2+y^2)^2 \, dy\,dx$

$0 \le r \le 5$
$0 \le \theta \le 2\pi$

$\frac{10}{12} \int_0^{2\pi} \int_0^5 r^5 \, dr\,d\theta$

$= \frac{10}{6\pi} \int_0^{2\pi} 3125 \, d\theta = \boxed{\frac{3125}{3\pi}}$

7. $\int_C \frac{10\sqrt{3}}{3} xyz \, ds$ C is $(0,0,0) \to (2,2,2)$

$x = 2t$
$y = 2t$
$z = 2t$

$= \int_0^1 \frac{10\sqrt{24}}{3} 8t^3 \, dt$

$= \frac{20\sqrt{6}}{3} \left(2t^4 \Big|_0^1\right) = \boxed{\frac{40\sqrt{6}}{3}}$

8. $\int_0^3 \int_{\sqrt{y}}^1 4e^{x^3} \, dx\,dy$

$4 \int_0^1 \int_0^{3x^2} e^{x^3} \, dy\,dx = 4 \int_0^1 3x^2 e^{x^3} \, dx$

$= 4\left(e^{x^3} \Big|_0^1\right) = 4(e-1)$

$= \boxed{4e-4}$

9. $\iiint_E \frac{10(x^2+y^2+z^2)}{4\pi}\, dV$

$x^2+y^2+z^2=25$

$0 \le \rho \le 5$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \pi$

$\frac{10}{4\pi}\int_0^{\pi}\int_0^{2\pi}\int_0^{5} \rho^4 \sin\varphi \, d\rho\, d\theta\, d\varphi$

$= \left(\frac{10}{4\pi}\right)\left(\int_0^5 \rho^4 d\rho\right)\left(\int_0^{2\pi} d\theta\right)\left(\int_0^{\pi} \sin\varphi \, d\varphi\right)$

$= \left(\frac{10}{4\pi}\right)(625)(2\pi)(1+1)$

$= 6250$

10. Find $\nabla \cdot F$ if $F = \langle \sin(x^2y), \sin(yz^2), \sin(x^2z^2) \rangle$

$= \frac{\partial}{\partial x}\sin(x^2y) + \frac{\partial}{\partial y}\sin(yz^2) + \frac{\partial}{\partial z}\sin(x^2z^2)$

$= 2xy\cos(x^2y) + z^2\cos(yz^2) + 2x^2z\cos(x^2z^2)$